Hybrid Beamforming for Millimeter-Wave ISAC System with Multi-static Cooperative Localization

Minghao Yuan*, Dongxuan He*, Hao Yin†, and Hua Wang*
*School of Information and Electronics, Beijing Institute of Technology, Beijing 100081, China
†Institute of China Electronic System Engineering, Beijing 100141, China
Email: minghaoyuan@bit.edu.cn, dongxuan_he@bit.edu.cn, yinhao@cashq.ac.cn, wanghua@bit.edu.cn

ICC 2025 - IEEE International Conference on Communications | 979-8-3315-0521-9/25/$31.00 ©2025 IEEE | DOI: 10.1109/ICC52391.2025.11160820

Abstract—**Beamforming is a key technique for achieving integrated sensing and communication (ISAC). However, most existing works focus on mono-static sensing, which can only provide limited sensing accuracy and range. In this paper, we investigate hybrid beamforming design for millimeter-wave (mmWave) multiple-input multiple-output (MIMO) ISAC system with multi-static cooperative localization, where one access point (AP) simultaneously transmits communication beams to serve multiple user equipments (UEs) and transmits a sensing beam towards a target, and other nearby APs perform cooperative localization on the target by estimating the angle-of-arrivals (AOAs) of received echo signals. To characterize the target localization accuracy, we derive the squared position error bound (SPEB) of AOA-based cooperative localization by using the equivalent Fisher information matrix (EFIM). Then, the hybrid beamforming design problem is formulated to minimize the SPEB of target localization, while satisfying the signal-to-interference-plus-noise ratio (SINR) requirements of individual communication UEs, transmit power budget, and constant modulus constraints. To solve the non-convex problem, a semidefinite relaxation (SDR)-based alternating optimization algorithm is proposed. Simulation results demonstrate that the proposed hybrid beamforming can achieve localization accuracy close to fully-digital beamforming and outperform the baseline schemes.**

Index Terms—**Integrated sensing and communication, cooperative localization, squared position error bound, hybrid beamforming, semidefinite relaxation.**

I. Introduction

Next generation wireless networks need to have high-quality communication and high-accuracy sensing capabilities for enabling various emerging applications, such as smart city and intelligent transportation. Integrated sensing and communication (ISAC) has been widely considered as a key technology for the sixth generation (6G) wireless networks [1].

Exploiting the spatial degrees of freedom provided by multi-antenna technology, beamforming can simultaneously achieve high-speed data transmission and high-accuracy target sensing [2]. In recent years, beamforming design for multiple-input multiple-output (MIMO) ISAC systems has attracted considerable research interest [2]–[4]. Specifically, in a multiuser MIMO ISAC system, beamforming was designed to match the sensing-only beampattern, while guaranteeing the signal-to-interference-plus-noise ratio (SINR) requirements of individual user equipments (UEs) [3]. Different from the beampattern design [2], [3], the Cramér-Rao bound (CRB) was adopted to characterize the target estimation performance and formulate the beamforming optimization [4].

Despite the extensive research progress, existing works mainly focus on mono-static sensing, whose sensing accuracy and range are limited. To break through the bottleneck of mono-static sensing, multi-static cooperative sensing has been considered as a promising solution [5]. Recently, there have been a few works investigating beamforming for multi-static cooperative ISAC systems [6], [7]. For instance, a networked MIMO ISAC system was studied in [6], where the coordinated transmit beamforming of multiple base stations (BSs) was designed to serve multiple UEs and perform joint target detection by exploiting the echo signals. In [7], the performance tradeoff between communication sum-rate and sensing beampattern matching error for a cell-free massive MIMO ISAC system was investigated.

Although the target detection [6] and beampattern design [7] have been studied in multi-static cooperative ISAC systems, to the best of our knowledge, beamforming for multi-static cooperative MIMO ISAC systems considering target localization has not been investigated yet, it is still an significant and open problem. Moreover, the aforementioned works [2]–[7] employ fully-digital beamforming, which leads to high hardware cost and power consumption, especially for millimeter-wave (mmWave) systems equipped with large-scale antenna arrays. By contrast, hybrid analog and digital beamforming achieves a flexible tradeoff between system performance and hardware complexity and has been considered as a feasible solution for mmWave systems.

In this paper, we investigate hybrid beamforming design for mmWave MIMO ISAC systems with multi-static cooperative localization. The main contributions of this paper are summarized as follows:

- We propose a multi-static mmWave MIMO ISAC system for achieving multiuser communication and target localization simultaneously. In this system, one access point (AP) acts as the ISAC transmitter to form communication beams to serve multiple UEs and to form a sensing beam towards a target, and other nearby APs act as the sensing receivers to cooperatively locate on the target by estimating the angle-of-arrivals (AOAs) of received echo signals.
- To characterize the target localization accuracy, we derive the squared position error bound (SPEB) of AOA-based multi-static cooperative localization. On this basis, we formulate the hybrid beamforming design problem to

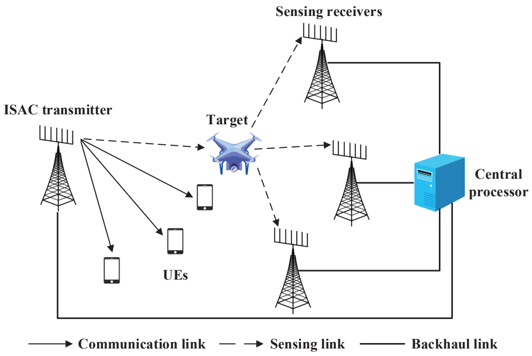

Fig. 1. System model.

minimize the SPEB of target localization, while satisfying the SINR requirements of individual UEs, transmit power budget, and constant modulus constraints.

- We propose a semidefinite relaxation (SDR)-based alternating optimization algorithm to address the above non-convex problem. Simulation results show that the proposed hybrid beamforming achieves localization accuracy close to fully-digital beamforming and significantly outperforms the baseline schemes.

II. System Model

We consider a multi-static mmWave MIMO ISAC system consisting of $M+1$ APs, where one AP acts as the ISAC transmitter, the other M APs act as the sensing receivers [8]. All the APs are connected to a central processor for information exchange and are assumed to be synchronized [6]. As shown in Fig. 1, the ISAC transmitter equipped with N_t antennas simultaneously forms communication beams to serve K single-antenna UEs and forms sensing beam towards a target, and the nearby M sensing receivers equipped with N_r antennas cooperatively locate the target by exploiting received echo signals. The cooperative passive sensing and data-level fusion [5] are considered in this system. Specifically, the M sensing receivers respectively estimate the individual AOAs of received echo signals and then transmit the estimation results of AOAs to the central processor for performing the cooperative localization on the target. The above processing is completed in a coherent processing interval, thus channel parameters and target parameters are assumed to be unchanged.

To reduce the hardware complexity and power consumption, we adopt the partially-connected hybrid beamforming [9]. Specifically, each AP is equipped with N_{RF} radio frequency (RF) chains, and each RF chain is connected to a subarray with N_t/N_{RF} antennas. Without loss of generality, uniform linear arrays (ULAs) with half-wavelength antenna spacing are deployed at all the APs.

A. Communication Model

For multiuser communication, the transmit signal after hybrid beamforming at time instant t can be expressed as

$$\mathbf{x}(t)=\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{s}(t)=\mathbf{F}_{\mathrm{RF}}\sum_{k=1}^{K}\mathbf{f}_{\mathrm{BB},k}s_k(t), \tag{1}$$

where $\mathbf{s}(t)=[s_1(t),\ldots,s_K(t)]\in\mathbb{C}^{K\times 1}$ is the transmitted communication symbol vector satisfying $\mathbb{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\}=\mathbf{I}_K$, $\mathbf{F}_{\mathrm{BB}}=[\mathbf{f}_{\mathrm{BB},1},\ldots,\mathbf{f}_{\mathrm{BB},K}]\in\mathbb{C}^{N_{\mathrm{RF}}\times K}$ denotes the digital beamformer, $\mathbf{F}_{\mathrm{RF}}\in\mathbb{C}^{N_t\times N_{\mathrm{RF}}}$ denotes the analog beamformer. Due to the partially-connected architecture, the analog beamformer is a block diagonal matrix, which can be represented as

$$\mathbf{F}_{\mathrm{RF}}=\mathrm{blkdiag}\left\{\mathbf{f}_1,\ldots,\mathbf{f}_{N_{\mathrm{RF}}}\right\}, \tag{2}$$

where $\mathbf{f}_i\in\mathbb{C}^{\frac{N_t}{N_{\mathrm{RF}}}\times 1}$ is the analog beamforming vector corresponding to the i-th subarray with each element having the constant modulus value, i.e., $|[\mathbf{f}_i]_j|=1, i=1,\ldots,N_{\mathrm{RF}}, j=1,\ldots,N_t/N_{\mathrm{RF}}$.

The received signal of the k-th UE can be expressed as

$$y_k(t)=\mathbf{h}_k^H\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{BB},k}s_k(t)+\sum_{j=1,j\neq k}^{K}\mathbf{h}_k^H\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{BB},j}s_j(t)+n_k(t), \tag{3}$$

where $\mathbf{h}_k\in\mathbb{C}^{N_t\times 1}$ represents the mmWave channel between the ISAC transmitter and the k-th UE, $n_k(t)\sim\mathcal{CN}(0,\sigma_n^2)$ is the additive white Gaussian noise (AWGN) with σ_n^2 denoting noise power. By adopting the extended Saleh-Valenzuela model [9], the mmWave channel between the ISAC transmitter and the k-th UE can be represented as

$$\mathbf{h}_k=\beta_k^{\mathrm{LOS}}\mathbf{a}_t\left(\phi_k^{\mathrm{LOS}}\right)+\sum_{l=1}^{L_k}\beta_{k,l}^{\mathrm{NLOS}}\mathbf{a}_t\left(\phi_{k,l}^{\mathrm{NLOS}}\right), \tag{4}$$

where β_k^{LOS} and ϕ_k^{LOS} denote the complex gain and azimuth angle-of-departure (AOD) of the line-of-sight (LOS) path, respectively, L_k is the number of non-line-of-sight (NLOS) paths, $\beta_{k,l}^{\mathrm{NLOS}}$ and $\phi_{k,l}^{\mathrm{NLOS}}$ denote the complex gain and azimuth AOD associated with the l-th NLOS path, respectively. The transmit steering vector can be represented as

$$\mathbf{a}_t(\phi)=\left[1,e^{j\frac{2\pi d}{\lambda}\sin\phi},\ldots,e^{j(N_t-1)\frac{2\pi d}{\lambda}\sin\phi}\right]^T, \tag{5}$$

where λ and d denote the signal wavelength and antenna spacing, respectively. It is assumed that channel state information is perfectly known.

The SINR of the k-th UE can be represented as

$$\mathrm{SINR}_k=\frac{\left|\mathbf{h}_k^H\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{BB},k}\right|^2}{\sum_{j=1,j\neq k}^{K}\left|\mathbf{h}_k^H\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{BB},j}\right|^2+\sigma_n^2}. \tag{6}$$

B. Sensing Model

For target sensing, it is assumed that there exists LOS path between the target and all the APs [2]–[9]. Thus, the received echo signal $\mathbf{y}_m(t)\in\mathbb{C}^{N_r\times 1}$ of the m-th sensing receiver at the t-th snapshot can be represented as

$$\begin{aligned}\mathbf{y}_m(t)&=\alpha_m\mathbf{A}_m\mathbf{x}(t)+\mathbf{n}_m(t)\\&=\alpha_m\mathbf{a}_r(\theta_m)\mathbf{a}_t^H(\theta_0)\mathbf{x}(t)+\mathbf{n}_m(t),\end{aligned} \tag{7}$$

where α_m is the reflection coefficient including both the distance dependent path loss and the radar cross section (RCS) of target, $\mathbf{n}_m(t)\sim\mathcal{CN}(0,\sigma_n^2\mathbf{I}_{N_r})$ denotes the AWGN vector, θ_0 is the azimuth AOD of the target relative to the ISAC transmitter, θ_m is the azimuth AOA of the target relative to the m-th sensing receiver, the steering vector is expressed as

$$\mathbf{a}(\theta) = \left[1, e^{j\frac{2\pi d}{\lambda}\sin\theta}, \ldots, e^{j(N_r-1)\frac{2\pi d}{\lambda}\sin\theta}\right]^T. \tag{8}$$

Based on (7), the received echo signal of the m-th sensing receiver over T snapshots can be expressed as

$$\mathbf{Y}_m = \alpha_m \mathbf{a}_r(\theta_m)\mathbf{a}_t^H(\theta_0)\mathbf{X} + \mathbf{N}_m, \tag{9}$$

where $\mathbf{Y}_m = [\mathbf{y}_m(1), \ldots, \mathbf{y}_m(T)]$, $\mathbf{N}_m = [\mathbf{n}_m(1), \ldots, \mathbf{n}_m(T)]$, $\mathbf{X} = [\mathbf{x}(1), \ldots, \mathbf{x}(T)]$. Therefore, the sample covariance matrix of transmit signal $\mathbf{X}$ over T snapshots can be represented as

$$\begin{aligned}\mathbf{R}_{\mathrm{X}} = \frac{1}{T}\mathbf{X}\mathbf{X}^H &= \frac{1}{T}\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{S}\mathbf{S}^H\mathbf{F}_{\mathrm{BB}}^H\mathbf{F}_{\mathrm{RF}}^H \\ &\approx \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^H\mathbf{F}_{\mathrm{RF}}^H.\end{aligned} \tag{10}$$

Note that the approximation in (10) tends to be accurate when the block length T is sufficiently large. Without loss of generality, this approximation will be regarded as an accurate equality in the rest of this paper [4].

III. SPEB Analysis and Problem Formulation

A. SPEB of AOA-based Cooperative Localization

Target localization is an important task in radar sensing. The target localization accuracy is typically measured by SPEB, which is derived from CRB. In this section, we first derive the equivalent Fisher information matrix (EFIM) of the AOAs, and then transform it into the SPEB of target localization.

In (9), the AOD θ_0 is typically known to the ISAC transmitter since it can be acquired from previous observations [2]. For the m-th sensing receiver, $\boldsymbol{\xi}_m = \left[\theta_m, \tilde{\boldsymbol{\alpha}}_m^T\right]^T \in \mathbb{R}^{3\times 1}$ denotes the vector of unknown parameters to be estimated, where $\tilde{\boldsymbol{\alpha}}_m = \left[\mathrm{Re}\{\alpha_m\}, \mathrm{Im}\{\alpha_m\}\right]^T$. The parameters of interest are all the AOAs, which are defined as $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_M]^T$.

In the following, we derive the EFIM of all the AOAs. Based on (9), the noiseless received echo signal can be vectorized as

$$\boldsymbol{\eta}_m = \alpha_m \mathrm{vec}(\mathbf{A}_m\mathbf{X}). \tag{11}$$

According to [10], the FIM for estimating $\boldsymbol{\xi}_m$ from the sufficient statistic $\boldsymbol{\eta}_m$ can be partitioned as

$$\mathbf{J}(\boldsymbol{\xi}_m) = \begin{bmatrix} J_{\theta_m\theta_m} & \mathbf{J}_{\theta_m\tilde{\boldsymbol{\alpha}}_m} \\ \mathbf{J}_{\theta_m\tilde{\boldsymbol{\alpha}}_m}^T & \mathbf{J}_{\tilde{\boldsymbol{\alpha}}_m\tilde{\boldsymbol{\alpha}}_m}\end{bmatrix}, \tag{12}$$

where

$$J_{\theta_m\theta_m} = \frac{2T|\alpha_m|^2}{\sigma_n^2}\mathrm{tr}\left(\dot{\mathbf{A}}_m\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^H\mathbf{F}_{\mathrm{RF}}^H\dot{\mathbf{A}}_m^H\right), \tag{13a}$$

$$\mathbf{J}_{\theta_m\tilde{\boldsymbol{\alpha}}_m} = \frac{2T}{\sigma_n^2}\mathrm{Re}\{\alpha_m^*\mathrm{tr}(\mathbf{A}_m\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^H\mathbf{F}_{\mathrm{RF}}^H\dot{\mathbf{A}}_m^H)[1,j]\}, \tag{13b}$$

$$\mathbf{J}_{\tilde{\boldsymbol{\alpha}}_m\tilde{\boldsymbol{\alpha}}_m} = \frac{2T}{\sigma_n^2}\mathrm{tr}(\mathbf{A}_m\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^H\mathbf{F}_{\mathrm{RF}}^H\mathbf{A}_m^H)\mathbf{I}_2, \tag{13c}$$

where $\dot{\mathbf{A}}_m = \frac{\partial \mathbf{A}_m}{\partial \theta_m}$. The equivalent Fisher information of θ_m can be expressed as

$$J_{\theta_m\theta_m}^e = J_{\theta_m\theta_m} - \mathbf{J}_{\theta_m\tilde{\boldsymbol{\alpha}}_m}\mathbf{J}_{\tilde{\boldsymbol{\alpha}}_m\tilde{\boldsymbol{\alpha}}_m}^{-1}\mathbf{J}_{\theta_m\tilde{\boldsymbol{\alpha}}_m}^T. \tag{14}$$

Therefore, the CRB for estimating θ_m can be given by

$$\mathrm{CRB}(\theta_m) = \left[J_{\theta_m\theta_m}^e\right]^{-1}. \tag{15}$$

Note that the AOAs of the target relative to different sensing receivers are independent, the EFIM of all the AOAs $\boldsymbol{\theta}$ can be expressed as

$$\mathbf{J}(\boldsymbol{\theta}) = \mathrm{diag}\left(J_{\theta_1\theta_1}^e, \ldots, J_{\theta_M\theta_M}^e\right). \tag{16}$$

Then, we analyze the SPEB of AOA-based cooperative localization. The positions of the target and the m-th sensing receiver are respectively denoted as $\mathbf{p} = [x, y]^T$ and $\mathbf{p}_m = [x_m, y_m]^T, m = 1, \ldots, M$. Therefore, the geometric relationship between the position of the target and the position of the m-th sensing receiver can be represented as

$$\theta_m = \arctan\frac{y - y_m}{x - x_m}. \tag{17}$$

The FIM of AOA-based cooperative localization $\mathbf{J}(\mathbf{p}) \in \mathbb{C}^{2\times 2}$ can be expressed as

$$\mathbf{J}(\mathbf{p}) = \mathbf{V}\mathbf{J}(\boldsymbol{\theta})\mathbf{V}^T = \sum_{m=1}^{M}\mathbf{v}_m J_{\theta_m\theta_m}^e \mathbf{v}_m^T, \tag{18}$$

where the Jacobian matrix $\mathbf{V} \in \mathbb{C}^{2\times M}$ is given by

$$\mathbf{V} = \frac{\partial \boldsymbol{\theta}^T}{\partial \mathbf{p}} = \begin{bmatrix} \frac{\partial\theta_1}{\partial x} \cdots \frac{\partial\theta_M}{\partial x} \\ \frac{\partial\theta_1}{\partial y} \cdots \frac{\partial\theta_M}{\partial y}\end{bmatrix}, \tag{19}$$

$\mathbf{v}_m = \frac{\partial\theta_m}{\partial\mathbf{p}}$ is the m-th column of $\mathbf{V}$. The SPEB of AOA-based cooperative localization can be represented as

$$\mathrm{SPEB} = \mathrm{tr}\left(\mathbf{J}(\mathbf{p})^{-1}\right). \tag{20}$$

It is observed from (13), (14), (18), and (20) that the SPEB is a function of hybrid beamforming matrix. Therefore, we can improve the localization accuracy by optimizing hybrid beamforming.

B. Problem Formulation

In this paper, we aim to jointly design the digital beamformer and partially-connected analog beamformer to minimize the SPEB of AOA-based target localization, while guaranteeing the SINR requirement of each UE, transmit power budget, and constant modulus constraints. The hybrid beamforming design problem can be formulated as

$$\min_{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}} \quad \mathrm{tr}\left(\mathbf{J}(\mathbf{p})^{-1}\right) \tag{21a}$$

$$\text{s.t.} \quad \frac{\left|\mathbf{h}_k^H\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{BB},k}\right|^2}{\sum_{j=1,j\neq k}^{K}\left|\mathbf{h}_k^H\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{BB},j}\right|^2 + \sigma_n^2} \geq \Gamma_k, \forall k, \tag{21b}$$

$$\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \leq P, \tag{21c}$$

$$\mathbf{F}_{\mathrm{RF}} \in \mathcal{A}_p, \tag{21d}$$

where Γ_k is the SINR threshold of the k-th UE, P is the power budget of the ISAC transmitter, $\mathcal{A}_p$ represents the feasible set of partially-connected analog beamformer in which constant modulus constraints are imposed on the non-zero elements of $\mathbf{F}_{\rm RF}$. Note that problem (21) is difficult to tackle due to the complicated objective function, highly coupled optimization variables, and non-convex constant modulus constraints.

IV. Hybrid Beamforming Design

In this section, we propose an SDR-based alternating optimization algorithm to design the hybrid beamforming.

A. Problem Reformulation

To tackle the complicated objective function (21a), we introduce an auxiliary variable $\mathbf{U} \in \mathbb{C}^{2\times 2}$ and the auxiliary variables $t_m, m = 1, \ldots, M$, which respectively satisfy

$$\sum_{m=1}^{M} \mathbf{v}_m J^e_{\theta_m\theta_m} \mathbf{v}_m^T \succeq \mathbf{U} \succeq \mathbf{0}, \tag{22a}$$

$$\mathrm{tr}(\dot{\mathbf{A}}_m \mathbf{R}_{\rm X} \dot{\mathbf{A}}_m^H) - \frac{|\mathrm{tr}(\mathbf{A}_m \mathbf{R}_{\rm X} \dot{\mathbf{A}}_m^H)|^2}{\mathrm{tr}\left(\mathbf{A}_m \mathbf{R}_{\rm X} \mathbf{A}_m^H\right)} \geq t_m, \forall m. \tag{22b}$$

Leveraging the monotonicity of function $\mathrm{tr}\left(\mathbf{A}^{-1}\right)$ on the positive semidefinite matrix space and the Schur complement condition, problem (21) can be equivalently reformulated as

$$\min_{\substack{\mathbf{F}_{\rm RF}, \mathbf{F}_{\rm BB}, \\ \mathbf{U}, t_m}} \ \mathrm{tr}\left(\mathbf{U}^{-1}\right) \tag{23a}$$

$$\text{s.t.} \ \sum_{m=1}^{M} \mathbf{v}_m \left(\frac{2T|\alpha_m|^2}{\sigma_n^2} t_m\right) \mathbf{v}_m^T \succeq \mathbf{U}, \mathbf{U} \succeq \mathbf{0}, \tag{23b}$$

$$\begin{bmatrix} \mathrm{tr}(\dot{\mathbf{A}}_m \mathbf{R}_{\rm X} \dot{\mathbf{A}}_m^H) - t_m & \mathrm{tr}(\mathbf{A}_m \mathbf{R}_{\rm X} \dot{\mathbf{A}}_m^H) \\ \mathrm{tr}(\dot{\mathbf{A}}_m \mathbf{R}_{\rm X} \mathbf{A}_m^H) & \mathrm{tr}\left(\mathbf{A}_m \mathbf{R}_{\rm X} \mathbf{A}_m^H\right) \end{bmatrix} \succeq \mathbf{0}, \forall m, \tag{23c}$$

$$\text{(21b)} - \text{(21d)}. \tag{23d}$$

B. Digital Beamformer Design

In this subsection, we optimize the digital beamformer $\mathbf{F}_{\rm BB}$ with given the analog beamformer $\mathbf{F}_{\rm RF}$. Exploiting the block diagonal structure of $\mathbf{F}_{\rm RF}$, the transmit power can be rewritten as $\|\mathbf{F}_{\rm RF}\mathbf{F}_{\rm BB}\|_F^2 = \frac{N_t}{N_{\rm RF}} \mathrm{tr}\left(\mathbf{F}_{\rm BB}\mathbf{F}_{\rm BB}^H\right)$. Thus, the subproblem with respect to $\mathbf{F}_{\rm BB}$ can be formulated as

$$\min_{\substack{\mathbf{F}_{\rm BB}, \\ \mathbf{U}, t_m}} \ \mathrm{tr}\left(\mathbf{U}^{-1}\right) \tag{24a}$$

$$\text{s.t.} \begin{bmatrix} \mathrm{tr}(\ddot{\mathbf{\Psi}}_m \mathbf{F}_{\rm BB}\mathbf{F}_{\rm BB}^H) - t_m & \mathrm{tr}(\dot{\mathbf{\Psi}}_m \mathbf{F}_{\rm BB}\mathbf{F}_{\rm BB}^H) \\ \mathrm{tr}(\dot{\mathbf{\Psi}}_m^H \mathbf{F}_{\rm BB}\mathbf{F}_{\rm BB}^H) & \mathrm{tr}\left(\mathbf{\Psi}_m \mathbf{F}_{\rm BB}\mathbf{F}_{\rm BB}^H\right) \end{bmatrix} \succeq \mathbf{0}, \forall m, \tag{24b}$$

$$(1 + \frac{1}{\Gamma_k}) \mathbf{f}_{{\rm BB},k}^H \tilde{\mathbf{H}}_k \mathbf{f}_{{\rm BB},k} \geq \sum_{j=1}^{K} \mathbf{f}_{{\rm BB},j}^H \tilde{\mathbf{H}}_k \mathbf{f}_{{\rm BB},j} + \sigma_n^2, \forall k, \tag{24c}$$

$$\frac{N_t}{N_{\rm RF}} \mathrm{tr}\left(\mathbf{F}_{\rm BB}\mathbf{F}_{\rm BB}^H\right) \leq P, \tag{24d}$$

$$\text{(23b)}, \tag{24e}$$

where $\ddot{\mathbf{\Psi}}_m \triangleq \mathbf{F}_{\rm RF}^H \dot{\mathbf{A}}_m^H \dot{\mathbf{A}}_m \mathbf{F}_{\rm RF}$, $\dot{\mathbf{\Psi}}_m \triangleq \mathbf{F}_{\rm RF}^H \dot{\mathbf{A}}_m^H \mathbf{A}_m \mathbf{F}_{\rm RF}$, $\mathbf{\Psi}_m \triangleq \mathbf{F}_{\rm RF}^H \mathbf{A}_m^H \mathbf{A}_m \mathbf{F}_{\rm RF}$, $\tilde{\mathbf{H}}_k \triangleq \mathbf{F}_{\rm RF}^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{F}_{\rm RF}$. It is observed that problem (24) is non-convex due to the quadratic constraint (24b) and SINR constraint (24c). We adopt the SDR technique [11] to tackle the above problem. Specifically, we define the auxiliary variables $\mathbf{W}_k = \mathbf{f}_{{\rm BB},k}\mathbf{f}_{{\rm BB},k}^H, \forall k$ such that $\mathbf{W}_k \succeq \mathbf{0}$ and $\mathrm{rank}\left(\mathbf{W}_k\right) = 1, \forall k$. By dropping the non-convex rank-one constraints, problem (24) can be relaxed as

$$\min_{\substack{\mathbf{W}_k, \\ \mathbf{U}, t_m}} \mathrm{tr}\left(\mathbf{U}^{-1}\right) \tag{25a}$$

$$\text{s.t.} \begin{bmatrix} \mathrm{tr}(\ddot{\mathbf{\Psi}}_m \sum_{k=1}^{K} \mathbf{W}_k) - t_m & \mathrm{tr}(\dot{\mathbf{\Psi}}_m \sum_{k=1}^{K} \mathbf{W}_k) \\ \mathrm{tr}(\dot{\mathbf{\Psi}}_m^H \sum_{k=1}^{K} \mathbf{W}_k) & \mathrm{tr}(\mathbf{\Psi}_m \sum_{k=1}^{K} \mathbf{W}_k) \end{bmatrix} \succeq \mathbf{0}, \forall m, \tag{25b}$$

$$(1 + \frac{1}{\Gamma_k}) \mathrm{tr}\left(\tilde{\mathbf{H}}_k \mathbf{W}_k\right) \geq \sum_{j=1}^{K} \mathrm{tr}\left(\tilde{\mathbf{H}}_k \mathbf{W}_j\right) + \sigma_n^2, \forall k, \tag{25c}$$

$$\mathrm{tr}\left(\sum_{k=1}^{K} \mathbf{W}_k\right) \leq \frac{P N_{\rm RF}}{N_t}, \tag{25d}$$

$$\mathbf{W}_k \succeq \mathbf{0}, \forall k, \tag{25e}$$

$$\text{(23b)}. \tag{25f}$$

It is observed that problem (25) is a convex semidefinite programming (SDP) problem, which can be efficiently solved by CVX. However, the solution to problem (25) may not be a feasible solution to the problem (24) due to the omission of the rank-one constraints. Therefore, we reconstruct the rank-one solution via eigenvalue decomposition [11].

C. Analog Beamformer Design

In this subsection, we optimize the analog beamformer $\mathbf{F}_{\rm RF}$ with given the digital beamformer $\mathbf{F}_{\rm BB}$. Leveraging the block diagonal structure, $\mathbf{F}_{\rm RF}$ can be rewritten as

$$\mathbf{F}_{\rm RF} = \tilde{\mathbf{F}}_{\rm RF} \mathbf{\Phi} = \mathrm{diag}\left(\mathbf{f}_{\rm RF}\right) \mathbf{\Phi}, \tag{26}$$

where $\tilde{\mathbf{F}}_{\rm RF} = \mathrm{blkdiag}\left\{\mathrm{diag}\left(\mathbf{f}_1\right), \ldots, \mathrm{diag}\left(\mathbf{f}_{N_{\rm RF}}\right)\right\} \in \mathbb{C}^{N_t \times N_t}$ is a diagonal matrix composed of the non-zero elements of $\mathbf{F}_{\rm RF}$, $\mathbf{\Phi} = \mathrm{blkdiag}\left\{\mathbf{1}, \mathbf{1}, \ldots, \mathbf{1}\right\} \in \mathbb{C}^{N_t \times N_{\rm RF}}$ is a block diagonal matrix and $\mathbf{1}$ is an $N_t/N_{\rm RF}$ dimensional vector with all elements being 1, $\mathbf{f}_{\rm RF} = [\mathbf{f}_1^T, \cdots, \mathbf{f}_{N_{\rm RF}}^T]^T \in \mathbb{C}^{N_t \times 1}$ is a vector consisting of the non-zero elements of $\mathbf{F}_{\rm RF}$. Thus, the transmit covariance matrix can be rewritten as

$$\begin{aligned} \mathbf{R}_{\rm X} &= \sum_{k=1}^{K} \mathrm{diag}\left(\mathbf{f}_{\rm RF}\right) \mathbf{\Phi} \mathbf{f}_{{\rm BB},k} \mathbf{f}_{{\rm BB},k}^H \mathbf{\Phi}^H \mathrm{diag}\left(\mathbf{f}_{\rm RF}\right)^H \\ &= \sum_{k=1}^{K} \mathrm{diag}\left(\mathbf{\Phi} \mathbf{f}_{{\rm BB},k}\right) \mathbf{f}_{\rm RF} \mathbf{f}_{\rm RF}^H \mathrm{diag}\left(\mathbf{\Phi} \mathbf{f}_{{\rm BB},k}\right)^H. \end{aligned} \tag{27}$$

Algorithm 1 SDR-based alternating optimization algorithm

1: **Initialization:** $\mathbf{F}_{\mathrm{RF}}^{(0)}$, iteration number $n=0$, convergence threshold ϵ.
2: **repeat**
3: Update $\mathbf{F}_{\mathrm{BB}}^{(n)}$ by solving problem (25);
4: Update $\mathbf{F}_{\mathrm{RF}}^{(n)}$ by solving problem (29);
5: $n=n+1$;
6: **until** The objective value of problem (21) is converged or the maximum number of iterations is reached.
7: **Output:** $\mathbf{F}_{\mathrm{RF}}$, $\mathbf{F}_{\mathrm{BB}}$.

Thus, the subproblem with respect to $\mathbf{f}_{\mathrm{RF}}$ can be rewritten as

$$\min_{\substack{\mathbf{f}_{\mathrm{RF}},\\ \mathbf{U},t_m}} \operatorname{tr}\left(\mathbf{U}^{-1}\right) \tag{28a}$$

$$\text{s.t.} \begin{bmatrix} \operatorname{tr}(\ddot{\mathbf{\Pi}}_m \mathbf{f}_{\mathrm{RF}}\mathbf{f}_{\mathrm{RF}}^H) - t_m & \operatorname{tr}(\dot{\mathbf{\Pi}}_m \mathbf{f}_{\mathrm{RF}}\mathbf{f}_{\mathrm{RF}}^H) \\ \operatorname{tr}(\dot{\mathbf{\Pi}}_m^H \mathbf{f}_{\mathrm{RF}}\mathbf{f}_{\mathrm{RF}}^H) & \operatorname{tr}\left(\mathbf{\Pi}_m \mathbf{f}_{\mathrm{RF}}\mathbf{f}_{\mathrm{RF}}^H\right) \end{bmatrix} \succeq \mathbf{0}, \forall m, \tag{28b}$$

$$(1+\frac{1}{\Gamma_k})\mathbf{f}_{\mathrm{RF}}^H \bar{\mathbf{H}}_{k,k}\mathbf{f}_{\mathrm{RF}} \geq \sum_{j=1}^{K} \mathbf{f}_{\mathrm{RF}}^H \bar{\mathbf{H}}_{k,j}\mathbf{f}_{\mathrm{RF}} + \sigma_n^2, \forall k, \tag{28c}$$

$$|[\mathbf{f}_{\mathrm{RF}}]_i| = 1, \forall i, \tag{28d}$$

$$\text{(23b)}, \tag{28e}$$

where $\ddot{\mathbf{\Pi}}_m \triangleq \sum_{k=1}^{K} \operatorname{diag}\left(\mathbf{\Phi}\mathbf{f}_{\mathrm{BB},k}\right)^H \dot{\mathbf{A}}_m^H \dot{\mathbf{A}}_m \operatorname{diag}\left(\mathbf{\Phi}\mathbf{f}_{\mathrm{BB},k}\right)$, $\dot{\mathbf{\Pi}}_m \triangleq \sum_{k=1}^{K} \operatorname{diag}\left(\mathbf{\Phi}\mathbf{f}_{\mathrm{BB},k}\right)^H \dot{\mathbf{A}}_m^H \mathbf{A}_m \operatorname{diag}\left(\mathbf{\Phi}\mathbf{f}_{\mathrm{BB},k}\right)$, $\mathbf{\Pi}_m \triangleq \sum_{k=1}^{K} \operatorname{diag}\left(\mathbf{\Phi}\mathbf{f}_{\mathrm{BB},k}\right)^H \mathbf{A}_m^H \mathbf{A}_m \operatorname{diag}\left(\mathbf{\Phi}\mathbf{f}_{\mathrm{BB},k}\right)$, and $\bar{\mathbf{H}}_{k,j} \triangleq \operatorname{diag}\left(\mathbf{\Phi}\mathbf{f}_{\mathrm{BB},j}\right)^H \mathbf{h}_k\mathbf{h}_k^H \operatorname{diag}\left(\mathbf{\Phi}\mathbf{f}_{\mathrm{BB},j}\right)$. It is observed that problem (28) is non-convex since constraints (28b), (28c), and (28d) are non-convex. Thus, the SDR technique [11] is applied to address problem (28). Specifically, we define the auxiliary variable $\mathbf{Q} = \mathbf{f}_{\mathrm{RF}}\mathbf{f}_{\mathrm{RF}}^H$ such that $\mathbf{Q} \succeq \mathbf{0}$ and $\operatorname{rank}(\mathbf{Q}) = 1$. By dropping the non-convex rank-one constraint, problem (28) can be relaxed as

$$\min_{\mathbf{Q},\mathbf{U},t_m} \operatorname{tr}\left(\mathbf{U}^{-1}\right) \tag{29a}$$

$$\text{s.t.} \begin{bmatrix} \operatorname{tr}(\ddot{\mathbf{\Pi}}_m \mathbf{Q}) - t_m & \operatorname{tr}(\dot{\mathbf{\Pi}}_m \mathbf{Q}) \\ \operatorname{tr}(\dot{\mathbf{\Pi}}_m^H \mathbf{Q}) & \operatorname{tr}\left(\mathbf{\Pi}_m \mathbf{Q}\right) \end{bmatrix} \succeq \mathbf{0}, \forall m, \tag{29b}$$

$$(1+\frac{1}{\Gamma_k})\operatorname{tr}\left(\bar{\mathbf{H}}_{k,k}\mathbf{Q}\right) \geq \sum_{j=1}^{K} \operatorname{tr}\left(\bar{\mathbf{H}}_{k,j}\mathbf{Q}\right) + \sigma_n^2, \forall k, \tag{29c}$$

$$\left|[\mathbf{Q}]_{i,i}\right| = 1, \forall i, \ \mathbf{Q} \succeq \mathbf{0}, \tag{29d}$$

$$\text{(23b)}. \tag{29e}$$

Problem (29) is a convex SDP problem, whose optimal solution can be obtained by CVX. However, the rank of the optimal solution of problem (29) may be larger than one. Therefore, we extract a feasible rank-one solution from the optimal solution of problem (29) by utilizing the Gaussian randomization [11].

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Number of APs $M+1$	4
Number of ISAC transmitter	1
Number of sensing receivers M	3
Number of antennas at each AP $N_t(N_r)$	32
Number of RF chains at each AP N_{RF}	4
Number of UEs K	3
Number of target	1
Carrier frequency f_c	28 GHz
Noise power σ_n^2	-90 dBm
Transmit power P	30 dBm
Communication SINR threshold Γ_k	10 dB
Number of snapshots T	30

The proposed SDR-based alternating optimization algorithm for solving hybrid beamforming design problem (21) is summarized in **Algorithm 1**.

V. SIMULATION RESULTS

In this section, we provide numerical results to verify the effectiveness of the proposed algorithm. Unless otherwise specified, the simulation parameters are set in Table I. The positions of the APs, UEs, and target are set as follows. The ISAC transmitter is located at (0m, 0m), and the sensing receivers are located at (100m, 0m), (0m, 100m), (100m, 100m), respectively. The UEs are randomly distributed on a circle with the distance of 50m away from the ISAC transmitter, and the target is randomly distributed within a circle with the center of (50m, 50m) and the radius of 20m. The bi-static pass loss model [8] and mmWave pass loss model [12] are adopted for sensing and communication, respectively.

We consider the following baseline schemes for comparison.

1) **SPEB-Min based FDBF:** We design the fully-digital beamforming (FDBF) by using the proposed SPEB minimization criterion and SDR technique.

2) **Beampattern Approx. based HBF:** The beampattern of hybrid beamforming (HBF) is designed to approximate the ideal beampattern [3] by alternately optimizing the digital beamformer and analog beamformer.

3) **SIMO:** The ISAC transmitter is equipped with single antenna and cannot achieve beamforming. The sensing receivers are the same as the other schemes.

For all the schemes, the MUSIC algorithm is adopted in each sensing receiver to estimate the AOA of target, then the estimated AOAs are transmitted to the central processor to cooperatively estimate the position of target by using the least square method. The target localization error is evaluated by the root mean square error (RMSE) between estimated position and real position. The position error bound (PEB) provides a lower bound for RMSE.

Fig. 2 presents the target localization accuracy versus the transmit power. We observe that the proposed SPEB-Min based hybrid beamforming achieves localization accuracy close to fully-digital beamforming and outperforms the beampattern approximation [3] based hybrid beamforming and SIMO, which proves that adopting the SPEB as the performance metric can

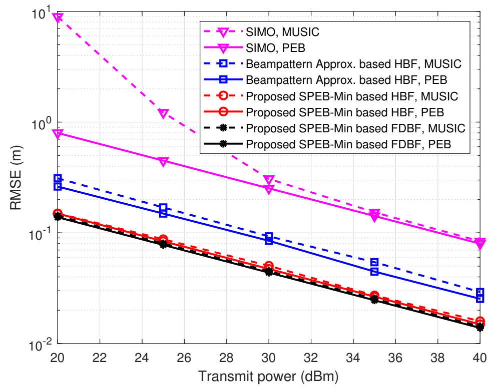

Fig. 2. The localization accuracy versus the tranmsit power, where $N_t = N_r = 32$, $N_{\rm RF} = 4$, $K = 3$, and $\Gamma_k = 10$ dB.

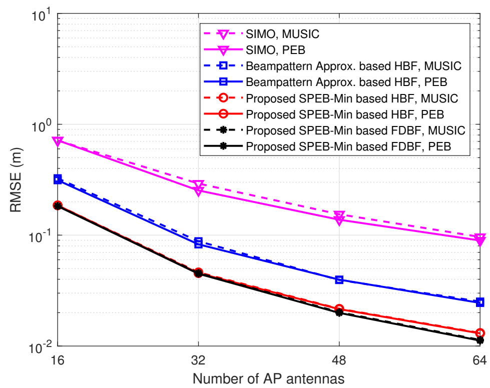

Fig. 3. The localization accuracy versus the number of AP antennas, where $N_t = N_r$, $N_{\rm RF} = 4$, and $K = 3$, $\Gamma_k = 10$ dB, and $P = 30$ dBm.

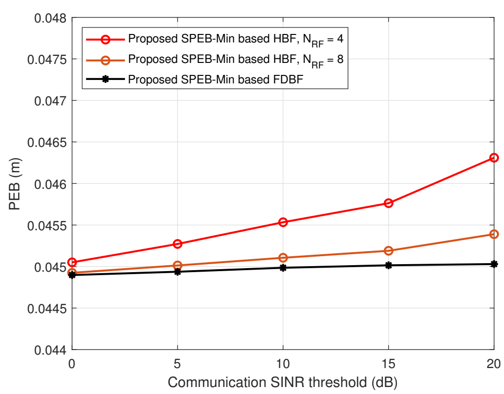

Fig. 4. The localization accuracy versus the communication SINR threshold, where $N_t = N_r = 32$, $K = 3$, and $P = 30$ dBm.

indeed enhance the target localization accuracy. As expected, the PEB of the proposed SPEB-Min based beamforming provides a tight lower bound for the RMSE of MUSIC-based localization, thereby verifying the effectiveness of the proposed scheme.

Fig. 3 shows the impact of the number of AP antennas on the target localization accuracy. It is observed that the localization accuracy of the proposed SPEB-Min based hybrid beamforming can approach that of fully-digital beamforming and outperforms the baseline schemes regardless of the number of antennas. Moreover, the localization accuracy improves as the number of antennas increases, since more antennas can provide larger spatial degrees of freedom for enhancing the localization accuracy.

Fig. 4 illustrates the impact of the communication SINR threshold of UEs on the target localization accuracy. It can be seen that the localization error increases as the SINR requirements of UEs increase, which reveals the performance tradeoff between communication and sensing under limited resources. Furthermore, the performance gap between hybrid beamforming and fully-digital beamforming reduces with the increase of the number of RF chains.

VI. Concultion

This paper investigated hybrid beamforming design for mmWave MIMO ISAC system with multi-static cooperative localization. The SPEB of AOA-based cooperative localization was derived to characterize the target localization accuracy. On this basis, the hybrid beamforming design problem was formulated as the SPEB minimization of target localization under the communication SINR requirements of UEs. We proposed an SDR-based alternating optimization algorithm to address the non-convex problem. Simulation results showed that the proposed hybrid beamforming achieved localization accuracy close to fully-digital beamforming and outperformed the baseline schemes.

References

[1] F. Liu et al., "Integrated sensing and communications: Toward dual functional wireless networks for 6G and beyond," *IEEE J. Sel. Areas Commun.*, vol. 40, no. 6, pp. 1728–1767, Jun. 2022.

[2] X. Liu, T. Huang, N. Shlezinger, Y. Liu, J. Zhou, and Y. C. Eldar, "Joint transmit beamforming for multiuser MIMO communications and MIMO radar," *IEEE Trans. Signal Process.*, vol. 68, pp. 3929–3944, Jun. 2020.

[3] F. Liu, C. Masouros, A. Li, H. Sun, and L. Hanzo, "MU-MIMO communications with MIMO radar: From co-existence to joint transmission," *IEEE Trans. Wireless Commun.*, vol. 17, no. 4, pp. 2755–2770, Apr. 2018.

[4] F. Liu, Y.-F. Liu, A. Li, C. Masouros, and Y. C. Eldar, "Cramér-Rao bound optimization for joint radar-communication beamforming," *IEEE Trans. Signal Process.*, vol. 70, pp. 240–253, 2022.

[5] Z. Wei et al., "Integrated sensing and communication enabled multiple base stations cooperative sensing towards 6G," *IEEE Netw.*, vol. 38, no. 4, pp. 207–215, Jul. 2024.

[6] G. Cheng, Y. Fang, J. Xu, and D. W. K. Ng, "Optimal coordinated transmit beamforming for networked integrated sensing and communications," *IEEE Trans. Wireless Commun.*, vol. 23, no. 8, pp. 8200-8214, Aug. 2024.

[7] W. Mao, Y. Lu, C. -Y. Chi, B. Ai, Z. Zhong, and Z. Ding, "Communication-sensing region for cell-free massive MIMO ISAC systems," *IEEE Trans. Wireless Commun.*, vol. 23, no. 9, pp. 12396-12411, Sep. 2024.

[8] G. Liu et al., "Cooperative sensing for 6G mobile cellular networks: Feasibility, performance and field trial," *IEEE J. Sel. Areas Commun.*, vol. 42, no. 10, pp. 2863-2876, Oct. 2024.

[9] X. Wang, Z. Fei, J. A. Zhang, and J. Xu, "Partially-connected hybrid beamforming design for integrated sensing and communication systems," *IEEE Trans. Commun.*, vol. 70, no. 10, pp. 6648–6660, Oct. 2022.

[10] I. Bekkerman and J. Tabrikian, "Target detection and localization using MIMO radars and sonars," *IEEE Trans. Signal Process.*, vol. 54, no. 10, pp. 3873–3883, Oct. 2006.

[11] Z. Luo, W. Ma, A. M. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.

[12] M. R. Akdeniz et al., "Millimeter wave channel modeling and cellular capacity evaluation," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1164–1179, Jun. 2014.