

2D Movable Antenna Assisted ISAC: Joint Beamforming and Antenna Position Optimization

(Invited Paper)

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Abstract—This paper studies a two-dimensional (2D) movable-antenna (MA) assisted integrated sensing and communications (ISAC) system. The base station serves multiple single-antenna users and senses one point target using a reconfigurable 2D MA array. The considered design exploits antenna-position adaptation and transmit beamforming to improve the sensing signal-to-noise ratio (SNR) under user signal-to-interference-plus-noise ratio (SINR), power-budget, antenna-region, and minimum-spacing constraints. Different from conventional fixed-array ISAC designs, the considered model explicitly characterizes the two-dimensional MA coordinates in both the multiuser communication channels and the round-trip sensing channel, which enables joint exploitation of beam-domain and geometry-domain degrees of freedom. The resulting problem is nonconvex because the beamforming vectors and MA coordinates are coupled in both the sensing objective and the communication constraints. An alternating optimization (AO) method is developed, where the beamforming subproblem is handled by semidefinite relaxation (SDR) and Gaussian randomization, while the MA-position subproblem is solved by a second-order successive convex approximation (SCA) procedure with inner approximations for the SINR and minimum-spacing constraints. Simulation results show that the proposed MA design provides higher sensing SNR than fixed-array, antenna-selection, and random-position benchmarks while maintaining the required communication quality.

Index Terms—Movable antenna, integrated sensing and communications, beamforming, antenna position optimization, alternating optimization.

I. INTRODUCTION

INTEGRATED sensing and communications (ISAC) has become an important candidate technology for future wireless networks, since communication links and radar sensing tasks can share spectrum, hardware, and signal-processing resources [1], [2]. In many ISAC scenarios, communication quality and sensing accuracy are both sensitive to spatial geometry [3]. The availability of line-of-sight (LoS) propagation, the separation among users, and the direction of the target all affect the achievable beamforming gain [4].

Conventional fixed-position antenna (FPA) arrays can only adjust the excitation weights after deployment, while their element coordinates remain unchanged. This limits the spatial degrees of freedom (DoF) available for adapting the array response to different communication and sensing tasks.

Movable antenna (MA) provides a direct way to reconfigure the array geometry by treating antenna positions as optimization variables. In [5], the antenna positions, transmit power, and receive combiner were jointly optimized for multiuser communication, showing that position adjustment can effectively improve channel conditions and reduce power consumption. To reduce the hardware and control overhead of independently moving all elements, a cross-linked movable array architecture was proposed in [6], where antenna movement and multiuser transmission were jointly designed to balance performance gain and implementation complexity. In [7], prototype experiments were conducted to verify the feasibility of MA systems, where the results demonstrated that position reconfiguration can bring practical performance improvement without increasing the number of radio-frequency (RF) chains.

The combination of MAs and ISAC has also attracted increasing attention. In [8], antenna positioning and dual-functional beamforming were jointly investigated under perfect and imperfect CSI. In [9], antenna positions and beamforming were optimized for MA-enabled ISAC, while [10] further studied the communication-sensing tradeoff. These studies verify the benefit of position-beamforming coordination. Different from them, this work does not regard the extension from one-dimensional or generic MA configurations to a 2D moving region as the sole contribution. Instead, we focus on a 2D multiuser MA-ISAC model in which both coordinates of each MA element are embedded into the multiuser communication channels and the round-trip sensing channel, and the resulting position-beamforming coupling is handled together with SINR and inter-element spacing constraints [11].

TABLE I
MAIN SYMBOLS

Symbol	Definition
$\mathcal{K}, K$	GU set and number of GUs
$\mathcal{M}, M$	MA-element set and number of MA elements
$\mathbf{p}, H_B$	Horizontal coordinate and altitude of the BS
$\mathbf{s}_k, \mathbf{q}$	Coordinates of GU k and the RT
$\mathbf{t}_m, \mathbf{t}$	Coordinate of MA m and stacked MA-position vector
$L, d_{\min}$	Side length of the moving region and minimum spacing
d_k, d_s	BS-GU and BS-RT distances
$\mathbf{u}_k, \mathbf{u}_s$	Direction vectors toward GU k and the RT
$\mathbf{h}_{c,k}, \mathbf{H}_s$	Communication channel and sensing channel
$\mathbf{w}_k, \mathbf{z}$	Beamformer for GU k and transmitted signal
$\Gamma_c, P_{\max}$	SINR threshold and transmit-power budget
γ_k, γ_r	Communication SINR and sensing SNR

In this paper, a fixed-BS 2D MA-assisted ISAC system is investigated. The main contributions are threefold. First, a 2D multiuser MA-ISAC model is established, where the MA coordinates are explicitly incorporated into both the BS-to-user channels and the round-trip BS-to-target sensing channel. Second, a sensing signal-to-noise ratio (SNR) maximization problem is formulated by jointly optimizing transmit beamforming and 2D MA positions under multiuser signal-to-interference-plus-noise ratio (SINR), transmit-power, movement-region, and minimum inter-element spacing constraints. Third, an alternating-optimization (AO)-based algorithm is developed, where semidefinite relaxation (SDR) with Gaussian randomization is used for beamforming optimization, and a second-order successive convex approximation (SCA) method is used to update MA positions via curvature-bounded lower approximations and an inner approximation of the spacing constraint. Numerical results verify the sensing SNR gain of the proposed design over fixed-array, antenna-selection, and random-position benchmarks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

For ease of reference, the main symbols used in the system model are summarized in Table I. Auxiliary variables introduced only for algorithmic derivations are omitted.

A. System Model

Consider a fixed-BS ISAC system with one BS, K single-antenna ground users (GUs), and one point radar target (RT). The BS is located at $\mathbf{p} = [x_p, y_p]^T$ with altitude H_B and is equipped with a 2D MA array. The positions of GU k , the RT, and the m th MA element are denoted by $\mathbf{s}_k = [x_k, y_k]^T$, $\mathbf{q} = [x_q, y_q]^T$, and $\mathbf{t}_m = [x_m, y_m]^T$ [12], respectively. Let $\mathcal{K} = \{1, \dots, K\}$, $\mathcal{M} = \{1, \dots, M\}$, and $\mathbf{t} = [\mathbf{t}_1^T, \dots, \mathbf{t}_M^T]^T$.

The MA positions are optimized for a quasi-static narrowband LoS channel snapshot with perfect channel state information (CSI). Practical factors such as positioning errors, movement latency, non-line-of-sight (NLoS) propagation, and CSI uncertainty may cause phase or channel mismatch; robust hardware-aware design is left for future work. The 3D distance between the BS and GU k is

$$d_k = \sqrt{\|\mathbf{s}_k - \mathbf{p}\|_2^2 + H_B^2}, \quad (1)$$

and the corresponding horizontal direction vector is

$$\mathbf{u}_k = \frac{\mathbf{s}_k - \mathbf{p}}{d_k}. \quad (2)$$

For a given direction vector $\mathbf{u} \in \mathbb{R}^{2 \times 1}$, the 2D MA steering vector is written as

$$\mathbf{a}(\mathbf{t}, \mathbf{u}) = \left[e^{j \frac{2\pi}{\lambda} \mathbf{t}_1^T \mathbf{u}}, \dots, e^{j \frac{2\pi}{\lambda} \mathbf{t}_M^T \mathbf{u}} \right], \quad (3)$$

where λ is the carrier wavelength and $\mathbf{t} = [\mathbf{t}_1^T, \dots, \mathbf{t}_M^T]^T$ collects all MA coordinates.

The channel power gain from the BS to GU k is modeled as [13]

$$\beta_{c,k} = \frac{G_t^{\text{BS}} G_r^{\text{GU}} \lambda^2}{(4\pi)^2 d_k^2}, \quad (4)$$

where G_t^{BS} and G_r^{GU} denote the BS transmit gain and GU receive gain. Since the BS is fixed, its velocity is $\mathbf{v}_B = \mathbf{0}$. With user velocity $\mathbf{v}_k$, the Doppler shift is

$$f_{c,k} = \frac{(\mathbf{v}_B - \mathbf{v}_k)^T (\mathbf{p} - \mathbf{s}_k)}{\lambda d_k}. \quad (5)$$

Thus, the BS-to-GU channel vector is given by

$$\mathbf{h}_{c,k} = \sqrt{\beta_{c,k}} e^{-j \frac{2\pi d_k}{\lambda}} e^{j 2\pi T_s f_{c,k}} \mathbf{a}(\mathbf{t}, \mathbf{u}_k) \in \mathbb{C}^{1 \times M}. \quad (6)$$

The scalar Doppler phase in (6) is kept for channel consistency, although it does not change the single-snapshot power metrics.

Similarly, the 3D BS-RT distance and horizontal direction vector are

$$d_s = \sqrt{\|\mathbf{q} - \mathbf{p}\|_2^2 + H_B^2}, \quad \mathbf{u}_s = \frac{\mathbf{q} - \mathbf{p}}{d_s}. \quad (7)$$

The round-trip sensing-channel power gain is

$$\beta_s = \frac{G_t^{\text{BS}} G_r^{\text{BS}} \sigma_{\text{RCS}} \lambda^2}{(4\pi)^3 d_s^4}, \quad (8)$$

where G_r^{BS} is the BS receive gain and σ_{RCS} is the target radar cross section. With target velocity $\mathbf{v}_q$, the sensing Doppler shift is

$$f_s = \frac{(\mathbf{v}_B - \mathbf{v}_q)^T (\mathbf{p} - \mathbf{q})}{\lambda d_s}. \quad (9)$$

The sensing channel matrix is expressed as

$$\mathbf{H}_s = \sqrt{\beta_s} e^{-j \frac{4\pi d_s}{\lambda}} e^{j 2\pi T_s f_s} \mathbf{a}_s^H \mathbf{a}_s, \quad (10)$$

where $\mathbf{a}_s = \mathbf{a}(\mathbf{t}, \mathbf{u}_s)$.

The BS transmits a dual-functional signal

$$\mathbf{z} = \sum_{k=1}^K \mathbf{w}_k b_k, \quad (11)$$

where $\mathbf{w}_k \in \mathbb{C}^{M \times 1}$ is the beamforming vector for GU k and b_k is the information symbol with $\mathbb{E}\{|b_k|^2\} = 1$. The received SINR of GU k is

$$\gamma_k = \frac{|\mathbf{h}_{c,k} \mathbf{w}_k|^2}{\sum_{i=1, i \neq k}^K |\mathbf{h}_{c,k} \mathbf{w}_i|^2 + \sigma_c^2}. \quad (12)$$

After averaging over mutually independent user symbols, the sensing SNR used in the optimization is

$$\gamma_r = \frac{\sum_{k=1}^K \mathbf{w}_k^H \mathbf{H}_s^H \mathbf{H}_s \mathbf{w}_k}{\sigma_r^2}. \quad (13)$$

B. Problem Formulation

The MA elements are constrained within a square region with side length L , and a minimum spacing $d_{\min}$ is imposed to reduce mutual coupling [14]. The fixed-BS joint design is formulated as

$$\text{P1: } \max_{\{\mathbf{w}_k\}, \{\mathbf{t}_m\}} \gamma_r \quad (14a)$$

$$\text{s.t. } \gamma_k \geq \Gamma_c, \quad \forall k \in \mathcal{K}, \quad (14b)$$

$$\sum_{k=1}^K \|\mathbf{w}_k\|_2^2 \leq P_{\max}, \quad (14c)$$

$$-L/2 \leq x_m \leq L/2, \quad \forall m \in \mathcal{M}, \quad (14d)$$

$$-L/2 \leq y_m \leq L/2, \quad \forall m \in \mathcal{M}, \quad (14e)$$

$$\|\mathbf{t}_m - \mathbf{t}_{m'}\|_2^2 \geq d_{\min}^2, \quad 1 \leq m < m' \leq M. \quad (14f)$$

In problem (P1), (14b) guarantees the required communication quality, (14c) limits the BS transmit power, (14d) and (14e) restrict antenna movement, and (14f) enforces element separation.

III. PROPOSED ALTERNATING OPTIMIZATION ALGORITHM

Problem (P1) is difficult to solve directly because the MA coordinates appear inside the complex exponential steering vectors, and the communication SINR constraints are coupled with all user beamformers. We therefore use an AO-based framework with two variable blocks, namely the transmit beamforming vectors and the MA positions.

A. Transmit Beamforming Optimization

For fixed MA positions, define

$$\Phi = \mathbf{H}_s^H \mathbf{H}_s, \quad \mathbf{H}_k = \mathbf{h}_{c,k}^H \mathbf{h}_{c,k}. \quad (15)$$

The beamforming subproblem is

$$\text{P2: } \max_{\{\mathbf{w}_k\}} \sum_{k=1}^K \mathbf{w}_k^H \Phi \mathbf{w}_k \quad (16a)$$

$$\text{s.t. } |\mathbf{h}_{c,k} \mathbf{w}_k|^2 - \Gamma_c \sum_{i=1, i \neq k}^K |\mathbf{h}_{c,i} \mathbf{w}_i|^2 \geq \Gamma_c \sigma_c^2, \quad \forall k, \quad (16b)$$

$$\sum_{k=1}^K \|\mathbf{w}_k\|_2^2 \leq P_{\max}. \quad (16c)$$

The constant factor $1/\sigma_r^2$ in (13) is omitted in (16a) since it does not affect the optimizer.

Let $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ with $\mathbf{W}_k \succeq 0$ and $\text{rank}(\mathbf{W}_k) = 1$. Then (P2) can be rewritten as

$$\text{P3: } \max_{\{\mathbf{W}_k\}} \sum_{k=1}^K \text{tr}(\Phi \mathbf{W}_k) \quad (17a)$$

$$\text{s.t. } \text{tr}(\mathbf{H}_k \mathbf{W}_k) - \Gamma_c \sum_{i=1, i \neq k}^K \text{tr}(\mathbf{H}_i \mathbf{W}_i) \geq \Gamma_c \sigma_c^2, \quad \forall k, \quad (17b)$$

$$\sum_{k=1}^K \text{tr}(\mathbf{W}_k) \leq P_{\max}, \quad (17c)$$

$$\mathbf{W}_k \succeq 0, \quad \forall k, \quad (17d)$$

$$\text{rank}(\mathbf{W}_k) = 1, \quad \forall k. \quad (17e)$$

By dropping the rank-one constraint (17e), the following SDR problem is obtained [15]:

$$\max_{\{\mathbf{W}_k\}} \sum_{k=1}^K \text{tr}(\Phi \mathbf{W}_k) \quad (18)$$

$$\text{s.t. } (17b), (17c), (17d).$$

The relaxed problem is a semidefinite program (SDP) and can be solved by CVX. If the obtained matrices are not rank one, Gaussian randomization is used to recover feasible beamforming vectors [16]. Specifically, for $\mathbf{W}_k^* = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H$, a random candidate is generated by

$$\bar{\mathbf{w}}_k = \mathbf{U}_k \mathbf{\Lambda}_k^{1/2} \mathbf{r}_k, \quad \mathbf{r}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M). \quad (19)$$

All candidate beamformers are scaled to satisfy the power budget. Among the candidates satisfying all SINR constraints, the one with the largest sensing objective is selected.

B. Movable Antenna Position Optimization

For fixed beamforming vectors, the MA-position subproblem is

$$\text{P4: } \max_{\tilde{\mathbf{t}}} g(\tilde{\mathbf{t}}) \quad (20a)$$

$$\text{s.t. } f_k(\tilde{\mathbf{t}}) \geq \Gamma_c \sigma_c^2, \quad \forall k \in \mathcal{K}, \quad (20b)$$

$$-L/2 \leq x_m \leq L/2, \quad -L/2 \leq y_m \leq L/2, \quad \forall m, \quad (20c)$$

$$\|\mathbf{t}_m - \mathbf{t}_{m'}\|_2^2 \geq d_{\min}^2, \quad 1 \leq m < m' \leq M, \quad (20d)$$

where $\tilde{\mathbf{t}} = [\mathbf{t}_1^T, \dots, \mathbf{t}_M^T]^T$,

$$g(\tilde{\mathbf{t}}) = \frac{1}{\sigma_r^2} \sum_{k=1}^K \mathbf{w}_k^H \mathbf{H}_s^H \mathbf{H}_s \mathbf{w}_k, \quad (21)$$

and

$$f_k(\tilde{\mathbf{t}}) = |\mathbf{h}_{c,k}(\tilde{\mathbf{t}}) \mathbf{w}_k|^2 - \Gamma_c \sum_{i=1, i \neq k}^K |\mathbf{h}_{c,i}(\tilde{\mathbf{t}}) \mathbf{w}_i|^2. \quad (22)$$

Both $g(\tilde{\mathbf{t}})$ and $f_k(\tilde{\mathbf{t}})$ are nonlinear functions of the MA positions. The spacing constraint is also nonconvex because it is a lower bound on a convex quadratic function.

Algorithm 1 AO for Joint Beamforming and MA Positioning

Require: Feasible $\mathbf{t}^{(0)}$, $\{\mathbf{w}_k^{(0)}\}$, I_{AO} , I_{SCA} , N_{rand} , ϵ
Ensure: $\mathbf{t}^*$, $\{\mathbf{w}_k^*\}$

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1: Set  $r = 0$  and evaluate  $J^{(0)} = \gamma_r(\mathbf{t}^{(0)}, \{\mathbf{w}_k^{(0)}\})$ .
2: repeat
3:   Update  $\{\mathbf{h}_{c,k}\}$  and  $\mathbf{H}_s$  using  $\mathbf{t}^{(r)}$ .
4:   Solve the SDR problem and recover  $\{\tilde{\mathbf{w}}_k\}$  by Gaussian randomization.
5:   Use  $\{\tilde{\mathbf{w}}_k\}$  if feasible and better; otherwise keep  $\{\mathbf{w}_k^{(r)}\}$ .
6:   Solve the SCA subproblem with fixed beamformers to obtain  $\tilde{\mathbf{t}}$ .
7:   Use  $\tilde{\mathbf{t}}$  if feasible and better; otherwise keep  $\mathbf{t}^{(r)}$ .
8:   Update  $J^{(r+1)}$  and set  $r = r + 1$ .
9: until  $|J^{(r)} - J^{(r-1)}|/|J^{(r-1)}| \leq \epsilon$  or  $r = I_{AO}$ 
10: Output  $\mathbf{t}^* = \mathbf{t}^{(r)}$ ,  $\mathbf{w}_k^* = \mathbf{w}_k^{(r)}$ .
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At the r th SCA iteration, a concave lower bound of the sensing objective is constructed around $\tilde{\mathbf{t}}^{(r)}$ as

$$\hat{g}^{(r)}(\tilde{\mathbf{t}}) = g(\tilde{\mathbf{t}}^{(r)}) + \nabla g(\tilde{\mathbf{t}}^{(r)})^T (\tilde{\mathbf{t}} - \tilde{\mathbf{t}}^{(r)}) - \frac{\delta^{(r)}}{2} \|\tilde{\mathbf{t}} - \tilde{\mathbf{t}}^{(r)}\|_2^2, \quad (23)$$

where $\delta^{(r)}$ is selected as a curvature upper bound of $g(\tilde{\mathbf{t}})$ over the current SCA step and can be enlarged by backtracking if necessary. The detailed gradient, Hessian, and lower-bound derivations are provided in Appendix A. For each user, the communication constraint is inner-approximated by

$$\hat{f}_k^{(r)}(\tilde{\mathbf{t}}) = f_k(\tilde{\mathbf{t}}^{(r)}) + \nabla f_k(\tilde{\mathbf{t}}^{(r)})^T (\tilde{\mathbf{t}} - \tilde{\mathbf{t}}^{(r)}) - \frac{\delta_k^{(r)}}{2} \|\tilde{\mathbf{t}} - \tilde{\mathbf{t}}^{(r)}\|_2^2, \quad (24)$$

where $\delta_k^{(r)}$ is selected as a curvature upper bound of $f_k(\tilde{\mathbf{t}})$ over the current SCA step and can be enlarged by backtracking if necessary. The minimum-spacing constraint is linearized by

$$2(\mathbf{t}_m^{(r)} - \mathbf{t}_{m'}^{(r)})^T (\mathbf{t}_m - \mathbf{t}_{m'}) - \|\mathbf{t}_m^{(r)} - \mathbf{t}_{m'}^{(r)}\|_2^2 \geq d_{\min}^2. \quad (25)$$

The SCA subproblem is therefore

$$\text{P5: } \max_{\tilde{\mathbf{t}}} \hat{g}^{(r)}(\tilde{\mathbf{t}}) \quad (26a)$$

$$\text{s.t. } \hat{f}_k^{(r)}(\tilde{\mathbf{t}}) \geq \Gamma_c \sigma_c^2, \quad \forall k, \quad (26b)$$

$$-L/2 \leq x_m \leq L/2, \quad -L/2 \leq y_m \leq L/2, \quad \forall m, \quad (26c)$$

$$(25), \quad 1 \leq m < m' \leq M. \quad (26d)$$

Since problem (P5) is a convex quadratically constrained quadratic program (QCQP), it can be efficiently solved by standard convex optimization tools such as CVX. After obtaining the optimal solution of (P5), the reference point is updated as $\tilde{\mathbf{t}}^{(r+1)} = \tilde{\mathbf{t}}^*$. Owing to the lower-bound construction in (23) and (24), and the inner approximation of the spacing constraint in (25), the solution obtained at each SCA iteration remains feasible for the original constraints in (P4).

C. Overall AO Procedure and Complexity

The overall AO procedure is summarized in Algorithm 1. Since the previous feasible solution is retained in each block update, the sensing SNR is nondecreasing and bounded by the finite transmit power and moving region. Thus, the objective sequence converges, although global optimality is not guaranteed. The main complexity is

$$\mathcal{O}(I_{AO} (I_{SDR} K^3 M^6 + N_{rand} K^2 M + I_{SCA} (2M)^3)), \quad (27)$$

where the SDR step dominates for large M .

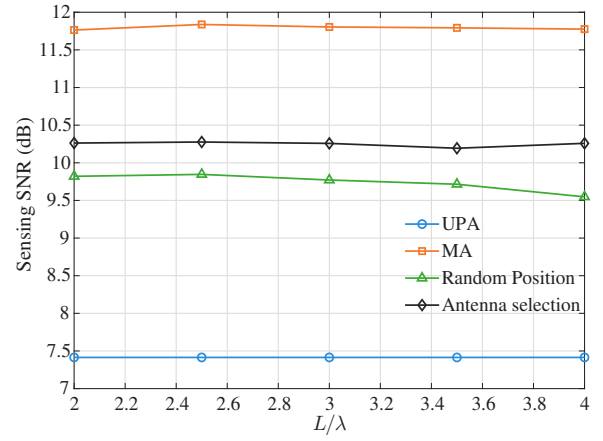


Fig. 1. Sensing SNR versus normalized movement region size.

IV. SIMULATION RESULTS

This section evaluates the proposed fixed-BS MA-ISAC design. Unless otherwise specified, the BS is located at the center of a $100 \text{ m} \times 100 \text{ m}$ region with altitude $H_B = 50 \text{ m}$, serving $K = 4$ GUs and sensing one RT with $M = 6$ MAs. The main parameters are $f_c = 3 \text{ GHz}$, $G_t^{\text{BS}} = 10 \text{ dBi}$, $G_r^{\text{BS}} = 7 \text{ dBi}$, $G_r^{\text{GU}} = 3 \text{ dBi}$, $\sigma_{\text{RCS}} = 0.2$, $P_{\max} = 30 \text{ dBm}$, $\sigma_c^2 = \sigma_r^2 = -80 \text{ dBm}$, $\Gamma_c = 10 \text{ dB}$, $L = 2\lambda$, and $d_{\min} = \lambda/2$. We set $N_{\text{rand}} = 5000$, $I_{AO} = 50$, $I_{SCA} = 25$, and $\epsilon = 10^{-4}$. All curves are averaged over 100 Monte Carlo realizations. Feasible UPA and random initializations are tested, and the best feasible result is reported.

The benchmarks include fixed uniform planar array (UPA), antenna selection from a UPA with $2M$ candidate elements, and random feasible MA positions. All schemes are evaluated under the same narrowband LoS and perfect-CSI setting.

Fig. 1 demonstrates the sensing SNR versus the normalized movement region size L/λ , where L denotes the side length of the feasible movement region for each MA and λ is the carrier wavelength. It can be observed that the proposed MA scheme consistently achieves the highest sensing SNR over the entire range. When L/λ increases from 2 to 4, the sensing SNR gain becomes marginal. This saturation does not imply that the 2D moving region is unnecessary; rather, it indicates that a compact region with $L = 2\lambda$ already provides sufficient phase-reconfiguration capability under the considered SINR and spacing constraints. Further enlarging the mechanical moving range brings little additional gain, which is favorable for practical implementation. In contrast, the UPA and antenna-selection schemes show limited variation since their antenna geometries cannot be continuously adjusted. The random-position scheme also provides lower performance due to the lack of optimized antenna placement.

Fig. 2 plots the sensing SNR versus the number of users, where $M = 10$ is used in this experiment. As K increases from 4 to 7, all schemes suffer a sensing-SNR loss because more transmit degrees of freedom are consumed by the multiuser SINR constraints. The proposed MA scheme still achieves the best performance, decreasing from about

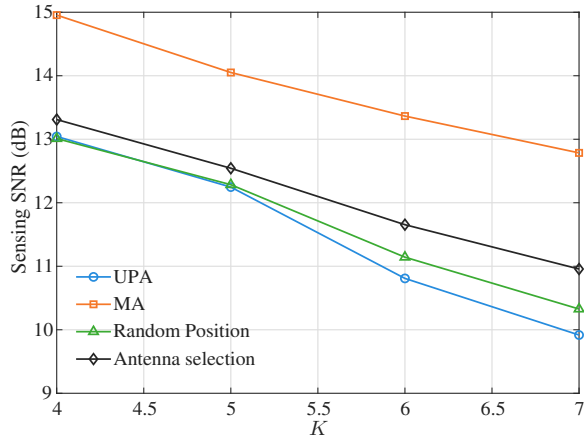


Fig. 2. Sensing SNR versus the number of users.

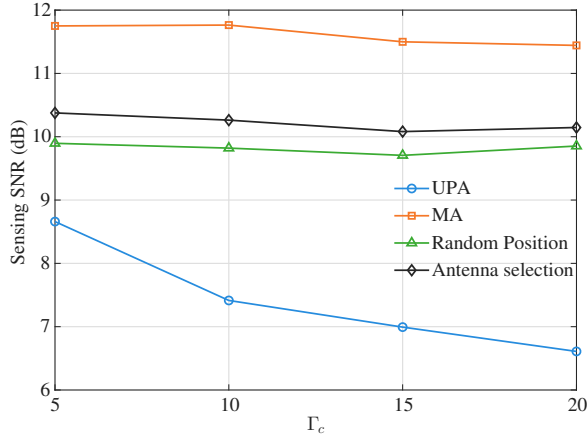


Fig. 3. Sensing SNR versus different communication SINR thresholds.

15 dB to about 12.8 dB. Compared with UPA, random position, and antenna selection, the proposed design preserves a clearer performance margin under heavy user loading. This result shows that optimizing the 2D antenna geometry can provide useful spatial flexibility for maintaining target-sensing performance while serving more users.

Fig. 3 presents the sensing SNR under different communication SINR thresholds. Increasing Γ_c imposes a stricter communication quality of service requirement, which forces the beamforming design to allocate more resources to user links and reduces the freedom left for sensing. The UPA scheme is the most sensitive to this constraint tightening, and its sensing SNR drops noticeably as Γ_c increases. In contrast, the proposed MA scheme exhibits only a mild degradation and remains the best among all schemes, with the sensing SNR staying above 11.4 dB over the tested threshold range. The antenna-selection and random-position schemes achieve intermediate performance, but their gains are limited by the absence of continuous position optimization. These results demonstrate that joint beamforming and MA-position optimization can better protect the sensing function under stringent communication requirements.

V. CONCLUSION

This paper investigated a fixed-BS 2D MA-assisted ISAC system. A sensing-SNR maximization problem was formulated by jointly optimizing transmit beamforming and MA positions under communication SINR, power, movement-region, and spacing constraints. An AO-based method was developed using SDR with Gaussian randomization and second-order SCA. Simulations showed that the proposed design outperforms UPA, antenna-selection, and random-position benchmarks. Future work will consider robust MA-position and beamforming optimization under imperfect CSI, NLoS propagation, positioning errors, and movement latency.

APPENDIX A

DERIVATIONS OF THE SCA APPROXIMATION

Let $\mu = 2\pi/\lambda$ and define $e_{q,m} = e^{j\mu \mathbf{t}_m^T \mathbf{u}_q}$, where $q = s$ denotes the sensing direction and $q = k$ denotes the direction of GU k . For compact notation, c_q denotes a link-dependent scalar coefficient, $\mathbf{B}_q$ denotes a Hermitian beamforming-related matrix, and $A_{q,m}$ is introduced only as an auxiliary scalar for derivative calculation. For a generic quadratic form

$$\phi_q(\tilde{\mathbf{t}}) = c_q \mathbf{a}_q \mathbf{B}_q \mathbf{a}_q^H, \quad (28)$$

where $\mathbf{B}_q = \mathbf{B}_q^H$, define

$$A_{q,m} = e_{q,m} \sum_{\ell=1}^M [\mathbf{B}_q]_{m,\ell} e_{q,\ell}^*. \quad (29)$$

Since $\partial e_{q,m} / \partial \mathbf{t}_m = j\mu e_{q,m} \mathbf{u}_q$, the gradient block with respect to $\mathbf{t}_m$ is

$$\nabla_{\mathbf{t}_m} \phi_q(\tilde{\mathbf{t}}) = -2c_q \mu \Im\{A_{q,m}\} \mathbf{u}_q. \quad (30)$$

Thus,

$$\nabla \phi_q(\tilde{\mathbf{t}}) = [\nabla_{\mathbf{t}_1} \phi_q, \dots, \nabla_{\mathbf{t}_M} \phi_q]^T. \quad (31)$$

The Hessian admits the following 2×2 block form:

$$\nabla^2 \phi_q(\tilde{\mathbf{t}}) = \begin{bmatrix} \mathbf{H}_{1,1}^{(q)} & \cdots & \mathbf{H}_{1,M}^{(q)} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{M,1}^{(q)} & \cdots & \mathbf{H}_{M,M}^{(q)} \end{bmatrix} \in \mathbb{R}^{2M \times 2M}, \quad (32)$$

where

$$\mathbf{H}_{m,\ell}^{(q)} = 2c_q \mu^2 \Re\{[\mathbf{B}_q]_{m,\ell} e_{q,m} e_{q,\ell}^*\} \mathbf{u}_q \mathbf{u}_q^T, \quad m \neq \ell, \quad (33)$$

and

$$\mathbf{H}_{m,m}^{(q)} = -2c_q \mu^2 \Re\{A_{q,m} - [\mathbf{B}_q]_{m,m}\} \mathbf{u}_q \mathbf{u}_q^T. \quad (34)$$

For the sensing objective $g(\tilde{\mathbf{t}})$ in (21), using $\mathbf{H}_s^H \mathbf{H}_s = M\beta_s \mathbf{a}_s^H \mathbf{a}_s$, we have

$$g(\tilde{\mathbf{t}}) = \frac{M\beta_s}{\sigma_r^2} \mathbf{a}_s \left(\sum_{i=1}^K \mathbf{w}_i \mathbf{w}_i^H \right) \mathbf{a}_s^H. \quad (35)$$

Therefore, the above gradient and Hessian formulas apply to $g(\tilde{\mathbf{t}})$ with

$$c_q = \frac{M\beta_s}{\sigma_r^2}, \quad \mathbf{B}_q = \sum_{i=1}^K \mathbf{w}_i \mathbf{w}_i^H, \quad \mathbf{u}_q = \mathbf{u}_s. \quad (36)$$

Accordingly, the Hessian of $g(\tilde{\mathbf{t}})$ can be written as

$$\nabla^2 g(\tilde{\mathbf{t}}) = \begin{bmatrix} \mathbf{H}_{1,1}^{(g)} & \cdots & \mathbf{H}_{1,M}^{(g)} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{M,1}^{(g)} & \cdots & \mathbf{H}_{M,M}^{(g)} \end{bmatrix}. \quad (37)$$

For the communication constraint function $f_k(\tilde{\mathbf{t}})$ in (22), since $\mathbf{h}_{c,k} = \alpha_{c,k} \mathbf{a}_k$ and $|\alpha_{c,k}|^2 = \beta_{c,k}$, we obtain

$$f_k(\tilde{\mathbf{t}}) = \beta_{c,k} \mathbf{a}_k \left(\mathbf{w}_k \mathbf{w}_k^H - \Gamma_c \sum_{i \neq k} \mathbf{w}_i \mathbf{w}_i^H \right) \mathbf{a}_k^H. \quad (38)$$

Hence, the above formulas apply to $f_k(\tilde{\mathbf{t}})$ with

$$c_q = \beta_{c,k}, \quad \mathbf{B}_q = \mathbf{w}_k \mathbf{w}_k^H - \Gamma_c \sum_{i \neq k} \mathbf{w}_i \mathbf{w}_i^H, \quad \mathbf{u}_q = \mathbf{u}_k. \quad (39)$$

The Hessian admits the block form

$$\nabla^2 f_k(\tilde{\mathbf{t}}) = \begin{bmatrix} \mathbf{H}_{1,1}^{(k)} & \cdots & \mathbf{H}_{1,M}^{(k)} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{M,1}^{(k)} & \cdots & \mathbf{H}_{M,M}^{(k)} \end{bmatrix} \in \mathbb{R}^{2M \times 2M}, \quad (40)$$

where $\mathbf{H}_{m,\ell}^{(k)}$ is obtained from $\mathbf{H}_{m,\ell}^{(q)}$ by substituting $c_q = \beta_{c,k}$, $\mathbf{B}_q = \mathbf{w}_k \mathbf{w}_k^H - \Gamma_c \sum_{i \neq k} \mathbf{w}_i \mathbf{w}_i^H$, and $\mathbf{u}_q = \mathbf{u}_k$.

At the r th SCA iteration, the curvature parameters can be chosen as

$$\delta^{(r)} \geq \left\| \nabla^2 g(\tilde{\mathbf{t}}^{(r)}) \right\|_2, \quad \delta_k^{(r)} \geq \left\| \nabla^2 f_k(\tilde{\mathbf{t}}^{(r)}) \right\|_2. \quad (41)$$

If necessary, they can be enlarged by backtracking over the current SCA step. Then, the quadratic lower bounds in (23) and (24) follow from the second-order Taylor lower-bound inequality. Moreover, since $\|\mathbf{t}_m - \mathbf{t}_{m'}\|_2^2$ is convex, its first-order Taylor expansion gives the inner approximation in (25).

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