

Beampattern Gain Optimization for Near-Field Wideband Multi-User ISAC Systems

(Invited Paper)

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Abstract—With the help of large-scale antenna arrays and sufficient bandwidth, integrated sensing and communication (ISAC) system can provide high-rate communications and high-accuracy target sensing. However, such ISAC system may experience nearfield propagation and severe beam squint effects, which makes the beamforming design challenging. To solve this problem, a near-field wideband ISAC system is investigated in this paper, which serves multiple communication users and sense one target simultaneously. To mitigate the adverse effect of beam squint, a phase-shifter (PS) and true-time-delay (TTD) architecture is adopted, which can construct a frequency-dependent analog beamformer. Then, an ISAC beamforming optimization problem is formulated to maximize the sensing beampattern gain while satisfying the communication SINR constraints. To handle the strong coupling among optimization variables, a penalty-based block coordinate descent (BCD) algorithm is developed. In particular, the auxiliary precoder is updated via successive convex approximation (SCA), and the PS-TTD analog beamformer is optimized through an ADMM-based method. Numerical results demonstrate that the proposed scheme effectively mitigates beam squint and achieves improved sensing beampattern performance while satisfying the communication QoS requirements.

Index Terms—Integrated sensing and communication (ISAC), near-field, beam squint, true-time-delay, hybrid beamforming.

I. INTRODUCTION

As one of the key technologies for sixth-generation (6G) wireless networks, integrated sensing and communication (ISAC) has been widely investigated in both academia and industry [1], [2]. To simultaneously realize high-rate communication and high-accuracy sensing, ISAC systems are expected to exploit extremely large-scale antenna arrays (ELAAs) and wide bandwidths [3]. However, the implementation of ELAA expands the near-field region, thus making the conventional far-field channel model inaccurate. In addition, wideband transmission suffers from severe beam squint effects, which leads to beam misalignment and performance degradation. Therefore, near-field wideband beamforming has become an important problem in ISAC systems.

Recently, near-field ISAC beamforming has attracted increasing attention. As a pioneering work, the authors in [4] proposed a beamforming design for both fully-digital and hybrid precoding architectures, which aims to minimize the sensing Cramér–Rao bound (CRB) under communication rate constraints. The authors in [5] considered a sensing-centric design, which can minimize the beampattern mean-square error (MSE) significantly. In [6], an squared position error bound (SPEB)-based hybrid beamforming design was investigated for near-field ISAC systems. Moreover, the authors in [7] investigated a multi-target detection scheme, in which the transmit power is minimized under both communication and sensing SINR constraints. However, most of the above studies are restricted to narrowband settings. In wideband near-field systems, different subcarriers generally exhibit different focusing directions and ranges due to the frequency-dependent array response, which is called the beam squint effect. Particularly, the beam squint effect degrade both sensing and communication performance obviously, which cannot be easily handled by the existing narrowband designs.

To mitigate the beam squint effect, true-time-delay (TTD) elements have been induced in to the wideband system, which can provide frequency-dependent phase compensation across different subcarriers [8], [9]. More specifically, TTD elements can mitigate beam squint and enable accurate beam focusing, thereby improving sensing accuracy. For example, the authors in [10] proposed a hybrid precoding design to match a prescribed beampattern across all subcarriers under communication SINR constraints, where ADMM is utilized to handle the coupling among hybrid precoding variables. Different from [10], the authors in [11] considered a hybrid analog digital (HAD) architecture, as well as a corresponding beamforming algorithm. However, both works are based on the far-field channel model. [12] considered a single-user near-field wideband ISAC system, where the Cramér–Rao bound

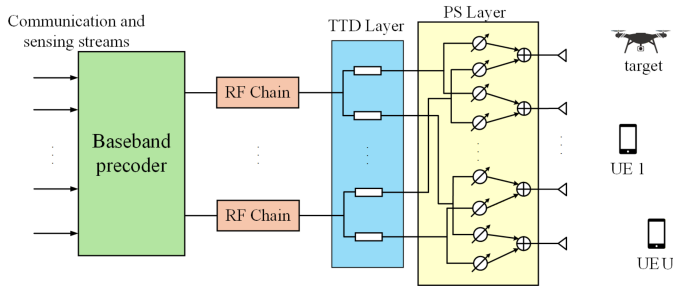


Fig. 1: Hybrid beamforming architecture and system model

minimization problem was reformulated as a weighted sum beampattern gain maximization problem. However, beamforming design for near-field wideband multi-user ISAC systems remains insufficiently explored.

Motivated by the aforementioned discussions, this paper investigates a near-field wideband multi-user ISAC beamforming design based on a PS-TTD hybrid architecture, where the sensing beampattern gain is maximized while the SINR requirements of multiple communication users are guaranteed on each subcarrier. To solve the above non-convex problem, a penalty-based block coordinate descent (BCD) algorithm is proposed. The auxiliary fully-digital precoder is updated via successive convex approximation (SCA). Additionally, we employ an ADMM-based method to decouple the strongly coupled PS coefficients and TTD delay variables. Numerical results demonstrate that the proposed scheme can effectively mitigate the wideband beam squint effect and improve the sensing beampattern performance while satisfying the communication QoS requirements.

Notations: Scalars, vectors, and matrices are denoted by lowercase, boldface lowercase, and boldface uppercase letters, respectively. $(\cdot)^H$, $(\cdot)^{-1}$, $\|\cdot\|_F$, $|\cdot|$, $\angle$ and $\mathbb{E}\{\cdot\}$ represent the conjugate transpose, the inverse, the Frobenius norm, the modulus, the phase and the expectation operator, respectively. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $\text{blkdiag}(\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_N)$ denotes a block diagonal matrix whose diagonal blocks are $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_N$. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ represent the real and imaginary parts of a complex number, respectively. c_0 denotes the speed of light.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

As illustrated in Fig.1, we consider a near-field wideband downlink ISAC system, where a base station (BS) equipped with N_t -element uniform linear array (ULA) serves U single-antenna users while simultaneously sensing a point target. To mitigate severe beam squint effect, the BS employs a PS-TTD hybrid beamforming architecture with N_{RF} RF chains, where each RF chain is connected to K TTDs and each TTD is connected to $Q = N_t/K$ PSs. The BS transmits an OFDM waveform with M subcarriers, where the total bandwidth is

B and carrier frequency is f_c . Accordingly, the frequency of the m -th subcarrier is given by

$$f_m = f_c + \frac{B}{M} \left(m - \frac{M+1}{2} \right), \quad m = 1, \dots, M. \quad (1)$$

For the proposed hybrid beamforming architecture, the frequency-dependent hybrid beamformer on the m -th subcarrier can be expressed as

$$\mathbf{F}_m = \Phi \mathbf{T}_m \mathbf{D}_m, \quad (2)$$

where $\mathbf{D}_m \in \mathbb{C}^{N_{\text{RF}} \times (U+1)}$ denotes the baseband digital precoder. $\Phi \in \mathbb{C}^{N_t \times K N_{\text{RF}}}$ denotes the frequency-independent analog beamforming matrix realized by the PS network, which can be expressed as

$$\Phi = \frac{1}{\sqrt{N_t}} [\Phi_1, \Phi_2, \dots, \Phi_{N_{\text{RF}}}], \quad (3)$$

where $\Phi_i = \text{blkdiag}(\phi_{i,1}, \phi_{i,2}, \dots, \phi_{i,K})$, and $\phi_{i,k} \in \mathbb{C}^{Q \times 1}$ denotes the phase-shifter vector of the Q PSs connected to the i -th RF chain through the k -th TTD. The frequency-dependent analog beamforming matrix realized by the TTD network on the m -th subcarrier is given by

$$\mathbf{T}_m = \text{blkdiag}(\mathbf{t}_{1,m}, \mathbf{t}_{2,m}, \dots, \mathbf{t}_{N_{\text{RF}},m}), \quad (4)$$

where

$$\mathbf{t}_{i,m} = [e^{-j2\pi f_m \tau_{i,1}}, e^{-j2\pi f_m \tau_{i,2}}, \dots, e^{-j2\pi f_m \tau_{i,K}}]^T, \quad (5)$$

and $\tau_{i,k}$ denotes the delay of the k -th TTD element connected to the i -th RF chain.

Assuming that the inter-element spacing of the ULA is $d = \lambda_c/2$, where $\lambda_c = c_0/f_c$ denotes the carrier wavelength. For a point located at polar coordinates (θ, r) with respect to the array center, the distance between the point and the n -th antenna element is given by

$$r_n(\theta, r) = \sqrt{r^2 + \delta_n^2 d^2 - 2r\delta_n d \sin \theta}, \quad (6)$$

where $\delta_n = n - \frac{N_t+1}{2}$, and $\delta_n d$ denotes the distance between the n -th antenna element and the center of the antenna array. Therefore, the near-field array response on the m -th subcarrier is expressed as

$$\mathbf{a}_m(\theta, r) = \frac{1}{\sqrt{N_t}} [e^{-j\psi_{1,m}}, e^{-j\psi_{2,m}}, \dots, e^{-j\psi_{N_t,m}}]^T, \quad (7)$$

where

$$\psi_{n,m}(\theta, r) \triangleq \frac{2\pi f_m}{c_0} (r_n(\theta, r) - r). \quad (8)$$

Based on the widely adopted Saleh-Valenzuela channel [13], the frequency-domain channel between the BS and the u -th user on the m -th subcarrier can be expressed as

$$\mathbf{h}_{u,m} = \sqrt{\frac{N_t}{L_u}} \sum_{l=1}^{L_u} \beta_{u,l,m} \mathbf{a}_m(\theta_{u,l}, r_{u,l}), \quad (9)$$

where L_u is the number of resolvable paths, $\beta_{u,l,m}$ denotes the complex gain of the l -th path on the m -th subcarrier, and $(\theta_{u,l}, r_{u,l})$ denotes the angular and range parameters of the l -th

path. Here, we assume the BS-user channel state information and target location are perfectly known.

Let $\mathbf{x}_m = \mathbf{F}_m \mathbf{s}_m = \Phi \mathbf{T}_m \mathbf{D}_m \mathbf{s}_m$ denote the transmitted signal on the m -th subcarrier, where $\mathbf{s}_m \in \mathbb{C}^{(U+1) \times 1}$ denotes the transmit symbol vector on the m -th subcarrier which satisfies $\mathbb{E}\{\mathbf{s}_m \mathbf{s}_m^H\} = \mathbf{I}_{(U+1)}$, where the additional data stream is dedicated to sensing. The received signal on the m -th subcarrier can be expressed as

$$y_{u,m} = \mathbf{h}_{u,m}^H \mathbf{x}_m + z_{u,m}, \quad (10)$$

where $z_{u,m} \sim \mathcal{CN}(0, \sigma_{u,m}^2)$ is the additive white Gaussian noise (AWGN). Thus, the SINR of the u -th user on the m -th subcarrier is given by

$$\gamma_{u,m} = \frac{|\mathbf{h}_{u,m}^H \mathbf{f}_{u,m}|^2}{\sum_{i \neq u}^{U+1} |\mathbf{h}_{u,m}^H \mathbf{f}_{i,m}|^2 + \sigma_{u,m}^2}, \quad (11)$$

where $\mathbf{f}_{u,m}$ is the u -th column of $\mathbf{F}_m$.

For sensing, we use the beampattern gain to characterize the performance. For a fixed point (θ_0, r_0) , the corresponding beampattern gain over all subcarriers is given by

$$g(\theta_0, r_0) = \sum_{m=1}^M \mathbf{a}_m^H(\theta_0, r_0) \mathbf{F}_m \mathbf{F}_m^H \mathbf{a}_m(\theta_0, r_0). \quad (12)$$

A larger $g(\theta_0, r_0)$ indicates stronger beam focusing toward the target point and is generally beneficial for sensing.

B. Problem Formulation

In this paper, we aim to jointly optimize the analog beamforming variables and digital precoders over all subcarriers. To maximize the beampattern gain while satisfying the SINR requirements for each communication user on every subcarrier, the resulting hybrid beamforming optimization problem can be formulated as

$$\max_{\Phi, \mathbf{T}_m, \mathbf{D}_m} g(\theta_0, r_0) \quad (13a)$$

$$\text{s.t. } \gamma_{u,m} \geq \Gamma_{u,m}, \quad \forall u, m, \quad (13b)$$

$$\|\mathbf{F}_m\|_F^2 \leq \frac{P_t}{M}, \quad \forall m, \quad (13c)$$

$$|[\Phi]_{i,j}| = \frac{1}{\sqrt{N_t}}, \quad \forall (i, j) \in \mathcal{S}, \quad (13d)$$

$$\tau_{i,k} \in [0, \tau_{\max}], \quad \forall i, k, \quad (13e)$$

where $\Gamma_{u,m}$ denotes the minimum SINR requirement of the u -th user on the m -th subcarrier, P_t denotes the maximum total transmit power over all subcarriers, $\mathcal{S}$ denotes the index set of nonzero entries in Φ , and $\tau_{\max}$ denotes the maximum realizable delay of the TTDs.

III. PROPOSED PENALTY-BASED BCD OPTIMIZATION

A. Problem Reformulation

The problem (13) is hard to solve due to the strong coupling among Φ , $\mathbf{T}_m$ and $\mathbf{D}_m$. To tackle this problem, we introduce

an auxiliary fully-digital precoder $\mathbf{W}_m$ and recast (13) into the following quadratic-penalty form, given by

$$\max_{\Phi, \mathbf{T}_m, \mathbf{D}_m, \mathbf{W}_m} \hat{g}(\mathbf{W}) - \frac{1}{\rho} \sum_{m=1}^M \|\mathbf{W}_m - \Phi \mathbf{T}_m \mathbf{D}_m\|_F^2 \quad (14a)$$

$$\text{s.t. } \hat{\gamma}_{u,m}(\mathbf{W}_m) \geq \Gamma_{u,m}, \quad \forall u, m, \quad (14b)$$

$$\|\mathbf{W}_m\|_F^2 \leq \frac{P_t}{M}, \quad \forall m, \quad (14c)$$

$$|[\Phi]_{i,j}| = \frac{1}{\sqrt{N_t}}, \quad \forall (i, j) \in \mathcal{S}, \quad (14d)$$

$$\tau_{i,k} \in [0, \tau_{\max}], \quad \forall i, k, \quad (14e)$$

where ρ is the penalty factor, and $\hat{\gamma}_{u,m}(\mathbf{W}_m)$ and $\hat{g}(\mathbf{W})$ denote the modified SINR and beampattern gain associated with $\mathbf{W}_m$, respectively, which are given by

$$\hat{\gamma}_{u,m}(\mathbf{W}_m) = \frac{|\mathbf{h}_{u,m}^H \mathbf{w}_{u,m}|^2}{\sum_{i \neq u}^{U+1} |\mathbf{h}_{u,m}^H \mathbf{w}_{i,m}|^2 + \sigma_{u,m}^2}. \quad (15)$$

and

$$\hat{g}(\mathbf{W}) = \sum_{m=1}^M \mathbf{a}_m^H(\theta_0, r_0) \mathbf{W}_m \mathbf{W}_m^H \mathbf{a}_m(\theta_0, r_0). \quad (16)$$

where $\mathbf{w}_{i,m}$ denotes the i -th column of $\mathbf{W}_m$.

As ρ decreases, the penalty term enforces the consistency between $\Phi \mathbf{T}_m \mathbf{D}_m$ and $\mathbf{W}_m$, thereby forcing the hybrid beamformer to approximate the auxiliary fully-digital precoder. We then employ a BCD-based algorithm, where each variable block is updated alternately while fixing the others until convergence.

B. Optimization for Φ and $\mathbf{T}_m$

With $\mathbf{W}_m$ and $\mathbf{D}_m$ fixed, the subproblem with respect to Φ and $\mathbf{T}_m$ only involves the penalty term, which can be written as

$$\min_{\Phi, \mathbf{T}_m} \sum_{m=1}^M \|\mathbf{W}_m - \Phi \mathbf{T}_m \mathbf{D}_m\|_F^2 \quad (17a)$$

$$\text{s.t. } (14d), (14e). \quad (17b)$$

However, due to the coupling between Φ and $\mathbf{T}_m$, the above problem is still non-convex. As such, an ADMM-based algorithm is further proposed. To decouple these two variables, we introduce an auxiliary variable $\mathbf{R}_m$ satisfying

$$\mathbf{R}_m = \Phi \mathbf{T}_m \quad \forall m. \quad (18)$$

Thus, the subproblem can be rewritten as

$$\min_{\mathbf{R}_m, \Phi, \mathbf{T}_m} \sum_{m=1}^M \|\mathbf{W}_m - \mathbf{R}_m \mathbf{D}_m\|_F^2 \quad (19a)$$

$$\text{s.t. } \mathbf{R}_m = \Phi \mathbf{T}_m, \quad \forall m, \quad (19b)$$

$$(14d), (14e). \quad (19c)$$

The equality constraint in (19b) is incorporated into the objective through the augmented Lagrangian, given by

$$\mathcal{L}_\eta = \sum_{m=1}^M \|\mathbf{W}_m - \mathbf{R}_m \mathbf{D}_m\|_F^2 + \frac{\eta}{2} \sum_{m=1}^M \left\| \mathbf{R}_m - \Phi \mathbf{T}_m + \frac{\Lambda_m}{\eta} \right\|_F^2. \quad (20)$$

Here Λ_m and η denote the unscaled dual variable and the penalty parameter introduced in the ADMM framework, respectively. Accordingly, the primal variables are updated by alternately minimizing $\mathcal{L}_\eta$ with respect to $\mathbf{R}_m$, Φ , and $\mathbf{T}_m$, subject to the constraints in (14d) and (14e).

1) *Update $\mathbf{R}_m$* : With the other variables fixed, the subproblem with respect to $\mathbf{R}_m$ is an unconstrained convex problem, given by

$$\begin{aligned} \mathbf{R}_m = \arg \min_{\mathbf{R}_m} & \|\mathbf{W}_m - \mathbf{R}_m \mathbf{D}_m\|_F^2 \\ & + \frac{\eta}{2} \left\| \mathbf{R}_m - \Phi \mathbf{T}_m + \frac{\Lambda_m}{\eta} \right\|_F^2 \\ & = \left(\mathbf{W}_m \mathbf{D}_m^H + \frac{\eta}{2} \mathbf{C}_m \right) \left(\mathbf{D}_m \mathbf{D}_m^H + \frac{\eta}{2} \mathbf{I}_{N_{\text{RF}}} \right)^{-1}, \end{aligned} \quad (21)$$

where $\mathbf{C}_m = \Phi \mathbf{T}_m - \Lambda_m/\eta$.

2) *Update Φ* : With the other variables fixed, the problem with respect to Φ is given by

$$\begin{aligned} \min_{\Phi} & \sum_{m=1}^M \left\| \mathbf{R}_m - \Phi \mathbf{T}_m + \frac{\Lambda_m}{\eta} \right\|_F^2 \\ \text{s.t.} & \text{ (14d)}. \end{aligned} \quad (22a) \quad (22b)$$

Define $\mathbf{V}_m = \mathbf{R}_m + \Lambda_m/\eta$. By exploiting the connection structure of the PS network, the objective function of (22) can be equivalently rewritten as

$$\begin{aligned} & \sum_{n=1}^{N_{\text{RF}}} \sum_{k=1}^K \sum_{m=1}^M \left(\phi_{n,k}^H \phi_{n,k} + \mathbf{v}_{m,n,k}^H \mathbf{v}_{m,n,k} \right. \\ & \left. - 2\Re \{ \mathbf{v}_{m,n,k}^H \phi_{n,k} e^{-j2\pi f_m \tau_{n,k}} \} \right), \end{aligned} \quad (23)$$

where $\mathbf{v}_{m,n,k} = \mathbf{V}_m [(k-1)Q + 1 : kQ, n]$. Since the first two terms in (23) are independent of the phase of $\phi_{n,k}$, minimizing (23) is equivalent to maximizing the last real-valued cross term. Therefore, the optimization of Φ can be decomposed into KN_{RF} independent subproblems. For each PS vector $\phi_{n,k}$, the corresponding subproblem is given by

$$\max_{\phi_{n,k}} \sum_{m=1}^M \Re \{ (\mathbf{v}_{m,n,k}^H \phi_{n,k} e^{-j2\pi f_m \tau_{n,k}}) \} \quad (24a)$$

$$\text{s.t. } |[\phi_{n,k}]_q| = 1, \quad q = 1, \dots, Q. \quad (24b)$$

The closed-form solution to (24) is

$$\phi_{n,k} = e^{j\angle(\sum_{m=1}^M \mathbf{v}_{m,n,k} e^{j2\pi f_m \tau_{n,k}})}. \quad (25)$$

3) *Update $\mathbf{T}_m$* : With the other variables fixed, updating $\mathbf{T}_m$ is equivalent to adjusting the delay variables $\tau_{n,k}$ of the TTDs. Therefore, the update of $\mathbf{T}_m$ can be performed by solving a set of one-dimensional search problems with respect to $\tau_{n,k}$. Denote the discrete search grid as $\mathcal{G}$. For each $\tau_{n,k}$, the grid-based solution is given by

$$\tau_{n,k} = \arg \max_{\tau_{n,k} \in \mathcal{G}} \sum_{m=1}^M \Re \{ \mathbf{v}_{m,n,k}^H \phi_{n,k} e^{-j2\pi f_m \tau_{n,k}} \}. \quad (26)$$

4) *Update Λ* : According to the ADMM framework [14], in the t -th ADMM iteration, the unscaled dual variable is updated as

$$\Lambda_m^{(t+1)} = \Lambda_m^{(t)} + \eta \left(\mathbf{R}_m^{(t+1)} - \Phi^{(t+1)} \mathbf{T}_m^{(t+1)} \right). \quad (27)$$

The penalty parameter η is fixed during the iterations.

C. Optimization for $\mathbf{D}_m$

With Φ , $\mathbf{T}_m$ and $\mathbf{W}_m$ fixed, the subproblem with respect to $\mathbf{D}_m$ is given by

$$\min_{\mathbf{D}_m} \|\mathbf{W}_m - \Phi \mathbf{T}_m \mathbf{D}_m\|_F^2, \quad (28)$$

which is an unconstrained least-squares problem with solution

$$\mathbf{D}_m = (\mathbf{T}_m^H \Phi^H \Phi \mathbf{T}_m)^{-1} \mathbf{T}_m^H \Phi^H \mathbf{W}_m. \quad (29)$$

After obtaining $\mathbf{D}_m$, it is scaled to satisfy the transmit power constraint, i.e.,

$$\mathbf{D}_m = \sqrt{\frac{P_t}{M \|\Phi \mathbf{T}_m \mathbf{D}_m\|_F^2}} \mathbf{D}_m. \quad (30)$$

D. Optimization for $\mathbf{W}_m$

With Φ , $\mathbf{T}_m$ and $\mathbf{D}_m$ fixed, the subproblem with respect to $\mathbf{W}_m$ can be formulated as

$$\max_{\mathbf{W}} \hat{g}(\mathbf{W}_m) - \frac{1}{\rho} \|\mathbf{W}_m - \mathbf{F}_m\|_F^2 \quad (31a)$$

$$\text{s.t. (14b), (14c)} \quad (31b)$$

Since $\hat{g}(\mathbf{W})$ is a convex quadratic function with respect to $\mathbf{W}_m$, maximizing it directly leads to a non-convex problem.

To make the subproblem tractable, the successive convex approximation (SCA) is adopted.

At the t -th iteration, the first-order Taylor expansion of $\hat{g}_m(\mathbf{W}_m)$ is given by

$$\begin{aligned} \hat{g}_m^{(t)}(\mathbf{W}) &= \hat{g}_m(\mathbf{W}^{(t)}) \\ &+ 2\Re \left\{ \text{tr} \left[\left(\mathbf{A}_m \mathbf{W}_m^{(t)} \right)^H \left(\mathbf{W}_m - \mathbf{W}_m^{(t)} \right) \right] \right\}, \end{aligned} \quad (32)$$

where $\mathbf{A}_m \triangleq \mathbf{a}_m(\theta_0, r_0) \mathbf{a}_m^H(\theta_0, r_0)$. Since $\hat{g}_m(\mathbf{W})$ is convex, $\hat{g}_m^{(t)}(\mathbf{W})$ is a global affine lower bound. Therefore, in the t -th SCA iteration, the original non-convex problem is approximated by

$$\max_{\mathbf{W}_m} \hat{g}_m^{(t)}(\mathbf{W}) - \frac{1}{\rho} \|\mathbf{W}_m - \mathbf{F}_m\|_F^2 \quad (33a)$$

$$\text{s.t. (15b), (15c)}. \quad (33b)$$

Algorithm 1: Penalty-Based BCD Algorithm

Input: $P_t, \theta_0, r_0, \Gamma_{u,m}, \mathbf{h}_{u,m}, \sigma_{u,m}, \forall u, m$.

1: Initialize $\Phi, \mathbf{T}_m, \mathbf{D}_m, \mathbf{W}_m, \rho > 0, 0 < c < 1$.

2: set $t \leftarrow 1$

3: **while** $t \leq T_{max}$ **do**

4: **repeat**

5: Update $\Phi, \mathbf{T}_m$ according to (25) and (26).

6: Update $\mathbf{D}_m$ according to (28) and (29).

7: Update $\mathbf{W}_m$ by solving (33).

8: **until** the inner BCD procedure converges;

9: Update penalty factor by $\rho \leftarrow c\rho$.

10: $t \leftarrow t + 1$.

11: **end**
Output: $\Phi, \mathbf{T}_m$ and $\mathbf{D}_m$.

Since the objective function in (33) is concave and the SINR constraints can be reformulated as standard second-order cone constraints [15], (33) is a convex optimization problem and can be efficiently solved by existing optimization tools.

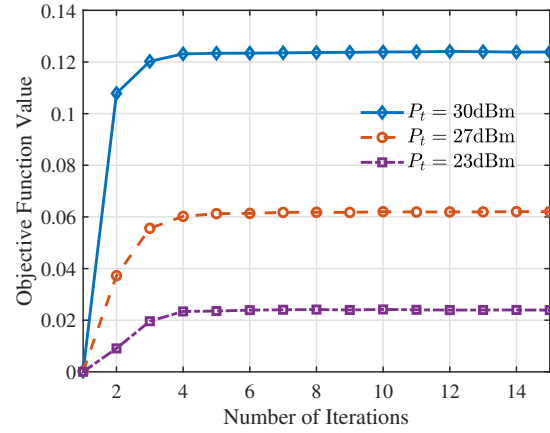
E. Overall Algorithm

Based on the above discussions, the overall penalty-based BCD algorithm is summarized in **Algorithm 1**. The convergence of **Algorithm 1** is analyzed through the objective sequence of the penalty-based problem. Specifically, the $\mathbf{W}_m$ -subproblem is handled by SCA with a tight lower-bound approximation, while the subproblems with respect to Φ , $\mathbf{T}_m$ and $\mathbf{D}_m$ reduce the hybrid approximation residual. Since the normalized beampattern gain is upper bounded by the transmit power constraint, the generated objective sequence is upper bounded. Therefore, the monotonic objective sequence converges.

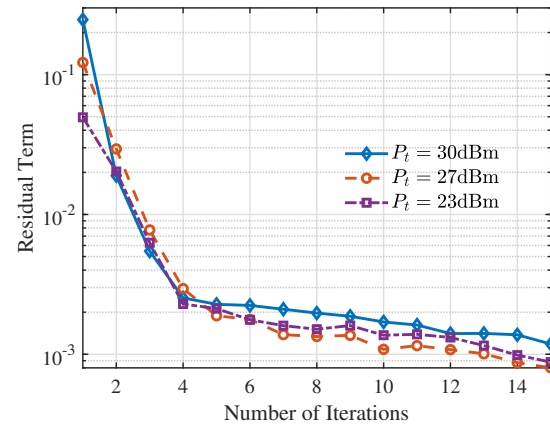
It can be shown that the complexities of updating Φ and $\mathbf{T}_m$, updating $\mathbf{D}_m$, and updating $\mathbf{W}_m$ can be roughly written as $\mathcal{O}(T_{\text{ADMM}}M(N_tN_{\text{RF}}(U+1) + (N_t+U+1)N_{\text{RF}}^2 + N_{\text{RF}}^3 + N_tN_{\text{RF}}|\mathcal{G}|))$, $\mathcal{O}(M(N_tN_{\text{RF}}(U+1) + (N_t+U+1)N_{\text{RF}}^2 + N_{\text{RF}}^3))$, and $\mathcal{O}(T_{\text{SCA}}M(N_t(U+1))^{3.5})$, respectively, where $\mathcal{O}(\cdot)$ is the big-O notation, T_{ADMM} and T_{SCA} denote the numbers of iterations required by the ADMM and SCA procedures, respectively.

IV. SIMULATION RESULTS

In this section, simulation results are presented to verify the effectiveness of the proposed algorithm. Unless otherwise specified, we assume that the BS is equipped with $N_t = 512$ antennas with half-wavelength inter-element spacing. The number of RF chains is $N_{\text{RF}} = 4$, where each RF chain is connected to $K = 8$ TTDs. The carrier frequency is $f_c = 30\text{GHz}$, the system bandwidth is $B = 3\text{GHz}$, and the number of subcarriers is $M = 8$. The system serves three communication users while simultaneously sensing one target region. The total transmit power is set to $P_t = 30\text{dBm}$. The SINR threshold for each communication user on each subcarrier is $\Gamma = 10\text{dB}$. The noise power over each subcarrier is $\sigma_{u,m}^2 = -90\text{dBm}$. The three communication users are located at $(-40^\circ, 20\text{m})$,



(a) Objective function



(b) Penalty term

Fig. 2: Convergence behaviors of the proposed algorithm

$(-15^\circ, 30\text{m})$, and $(15^\circ, 35\text{m})$, respectively. Besides, the center of the sensing region is located at $(-15^\circ, 10\text{m})$. The angle and distance ranges are set to $[-60^\circ, 60^\circ]$ and $[2, 30]\text{m}$, respectively. A three-path near-field channel model is adopted for each communication user. The line-of-sight (LoS) path is determined by the user location and includes large-scale path loss, while the angles and distances of the scattering paths are randomly and uniformly generated around the LoS path. The maximum TTD delay is set to $\tau_{\text{max}} = 4.25\text{ns}$, and the delay search grid contains 400 points. For the ADMM-based PS-TTD update, the initial penalty parameter is set to 10^{-4} . For the outer penalty-based BCD algorithm, the initial penalty parameter is set to 10^3 , with an update factor of $c = 0.6$.

For comparison, two benchmarks are considered, namely, fully-digital beamforming and PS-only beamforming. For the fully-digital beamforming, the beamformer is directly optimized without the PS-TTD hybrid hardware constraints, which serves as a performance upper bound. For the PS-only beamforming, the TTD network is disabled by setting the maximum delay to zero, i.e., $\tau_{\text{max}} = 0\text{ns}$.

Fig.2 demonstrates the convergence of the proposed algorithm. Fig.2(a) shows that the objective value increases mono-

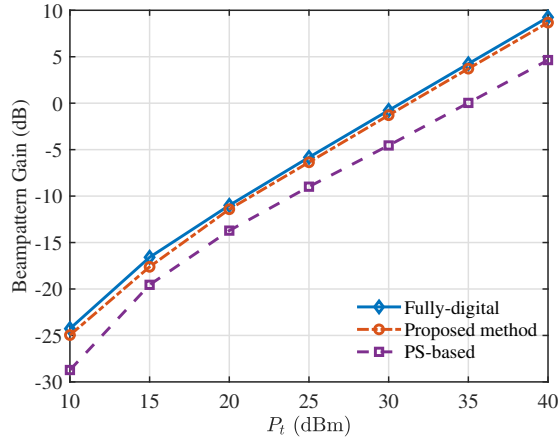


Fig. 3: Beampattern gain versus transmit power

tonically and converges within a few iterations, while Fig.2(b) shows that the penalty term gradually decreases, indicating the numerical convergence of the proposed algorithm.

Fig.3 demonstrates the beampattern gain achieved by the different schemes. It can be seen that the beampattern gain of all schemes increases as P_t increases, which is because a larger transmit power provides more available energy for sensing beam focusing. Furthermore, the proposed PS-TTD architecture achieves performance close to that of the fully-digital beamforming scheme over the whole transmit power range, indicating that the introduced TTD network can effectively compensate for the frequency-dependent beam squint effect. In contrast, the PS-only scheme suffers from performance degradation, due to the fact that frequency-independent analog beamformer cannot adequately align the beams over different subcarriers, resulting in the beams at subcarriers far away from the central subcarrier to steer toward unintended directions.

Fig.4 illustrates the beampattern gain versus the signal bandwidth. It can be observed that, since the fully-digital scheme can realize a frequency dependent beamforming matrix to mitigate beam split effect, its performance maintains almost unchanged performance over the considered bandwidth range. In contrast, the PS-only scheme suffers from noticeable performance degradation because of its frequency-independent analog beamformer. The above observations further verify the effectiveness of TTD elements in wideband beamforming in terms of mitigating the beam split effect.

V. CONCLUSIONS

In this paper, we proposed a beampattern gain optimization for a multi-user near-field wideband ISAC system with a PS-TTD hybrid beamforming architecture. To address the coupled and non-convex optimization problem, a penalty-based BCD algorithm was proposed, where an ADMM-based update was utilized to optimize the analog PS-TTD variables. Numerical results demonstrated the convergence and effectiveness of the proposed algorithm. In particular, the proposed PS-TTD architecture outperforms the conventional PS-only scheme,

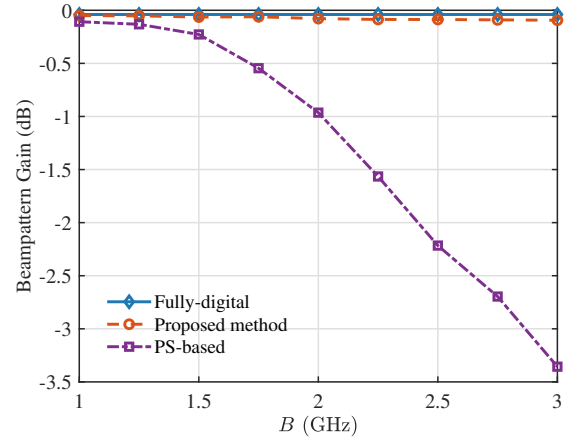


Fig. 4: Beampattern gain versus signal bandwidth

which indicates the effectiveness of TTDs in mitigating the wideband beam squint effect.

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