

VB-CPD-Based ISAC: Joint Tensor Rank and Parameter Estimation

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Abstract—Integrated sensing and communication (ISAC) systems can realize both communication and sensing functionalities. However, it is challenging to sense multiple targets simultaneously due to the coupling between parameters, which require more efficient tools such as tensor-based parameter estimation to obtain the information of targets. Moreover, traditional tensor-based parameter estimation methods typically rely on fixed rank prerequisites, where the uncertainty in tensor rank inevitably leads to severe performance degradation due to overfitting or underfitting. To solve this problem, a unified multi-dimensional tensor modeling and parameter estimation framework is proposed in this paper. More specifically, we first develop a Variational Bayesian Canonical Polyadic Decomposition (VB-CPD) algorithm. By incorporating hierarchical sparsity-inducing priors, the proposed VB-CPD algorithm can autonomously determine the effective tensor rank and prune redundant components directly from highly coupled observations, thus eliminating the need for prior knowledge of the numbers of users or targets. Furthermore, an alternating parameter decoupling (APD) algorithm is proposed to sequentially resolve the highly nonlinear coupling among spatial angles, time delays, and Doppler shifts, thereby enabling accurate physical parameter extraction. Simulation results demonstrate that the proposed framework outperforms conventional tensor decomposition baselines in both rank identification and parameter estimation accuracy.

Index Terms—Integrated sensing and communication, tensor decomposition, variational Bayesian inference, parameter estimation.

I. INTRODUCTION

Integrated Sensing and Communication (ISAC) has emerged as a key technology for future wireless networks, which seamlessly integrates radar sensing and communication capabilities into a unified platform [1], [2]. To achieve high-resolution sensing and high-rate transmission, millimeter-wave (mmWave)-based ISAC has attracted significant attention. By leveraging massive multiple-input multiple-output (MIMO) architectures in mmWave systems, the severe propagation path loss can be effectively compensated [3], thereby improving the communication and sensing performance significantly.

However, the sensing of ISAC system is still challenging, especially in the face of multiple targets [4]. In such scenarios, the aggregated ISAC signals inherently include high-

dimensional information, such as the spatial, temporal, and frequency domains, thus making the parameter estimation difficult [5]. To naturally represent and process this multi-dimensional data while preserving its underlying structural integrity, tensor has emerged as an effective mathematical tool [6], [7]. For example, The authors in [8] pioneered a unified tensor framework for massive MIMO-ISAC systems, leveraging the Vandermonde structure to accurately estimate channel state information and target parameters. The authors in [9] proposed a unified Canonical Polyadic decomposition (CPD)-based tensor framework, which can handle the signal coupling in large-scale access ISAC systems, thus enabling efficient active user detection and accurate parameter estimation simultaneously. Moreover, the work in [10] designed a tensor-based scheme specifically tailored for high-mobility grant-free random access scenarios, which effectively combats the severe Doppler shifts and time-varying fading inherent in highly dynamic environments.

However, the number of users or targets fluctuates continuously in practical scenarios, which remains completely unknown to the ISAC system. Moreover, an incorrectly predefined rank inevitably leads to severe overfitting or underfitting, which drastically degrades the parameter estimation accuracy. To be noticed, how to determine the tensor rank from noisy observations is generally an NP-hard problem. To solve this problem, Bayesian inference provides a flexible probabilistic paradigm. In particular, with the help of hierarchical sparsity-inducing priors, Bayesian inference can infer tensor rank accurately from highly coupled observations, thereby facilitating simultaneous accurate rank determination and joint parameter estimation [11].

Against this background, a robust parameter estimation framework based on Bayesian inference and tensor decomposition is proposed. The main contributions are summarized as follows: 1) The multi-dimensional ISAC signals are formulated as a third-order observation tensor, and a Variational Bayesian CPD (VB-CPD) algorithm is proposed. 2) Based on the robust factor matrices extracted via VB-CPD, an alternating decoupling and parameter extraction is proposed.

Notations: Scalars, vectors, matrices, and tensors are denoted by standard lowercase, boldface lowercase, boldface

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uppercase, and Euler script letters, respectively. Superscripts $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$, $(\cdot)^{-1}$, and $(\cdot)^\dagger$ denote the transpose, conjugate transpose, complex conjugate, inverse, and Moore-Penrose pseudo-inverse, respectively. The operators $\circ$, $\otimes$, and $\odot$ represent the outer, Kronecker, and Khatri-Rao products, respectively. The Frobenius and ℓ_2 norms are defined as $\|\cdot\|_F$ and $\|\cdot\|_2$. $\text{Re}\{\cdot\}$ extracts the real part of a complex number, and $\text{diag}(\mathbf{a})$ constructs a diagonal matrix from vector $\mathbf{a}$. $\mathbf{I}_N$ indicates the $N \times N$ identity matrix. $\llbracket \cdot \rrbracket$ and $\mathbb{E}[\cdot]$ denote the Kruskal operator and the mathematical expectation, $\mathcal{CN}(0, \sigma^2)$ represents a circularly symmetric complex Gaussian distribution with zero mean and variance σ^2 .

II. SYSTEM MODEL

In this paper, a mmWave massive MIMO ISAC system is considered, which includes an ISAC base station (BS) equipped with N_{BS} antennas and M_{BS} radio frequency (RF) chains. Moreover, the antennas and RF chains are arranged in two uniform linear arrays (ULAs) for transmission and reception, respectively. The BS receives uplink signals from U user equipments (UEs) and utilizes downlink ISAC signals to detect K targets concurrently. Each UE is equipped with N_{UE} antennas and M_{UE} RF chains. In this paper, we assume that the channel between the BS and target only experiences the line-of-sight (LoS) path, while the multipath effects are neglected. Besides, the OFDM signal with M subcarriers is employed. Each UE is assigned a unique precoding matrix $\mathbf{P}_u$ to enable identity-based parameter association at the receiver.

A. Channel Model

For communication, the frequency response matrix of channel between the u -th UE and the BS at the m -th subcarrier and the n -th OFDM symbol can be modeled as

$$\mathbf{H}_u^m(n) = \alpha_{c,u} e^{-j2\pi\tau_{c,u} f_{\text{samp}} \frac{m}{M}} \mathbf{a}_R(\theta_u) \mathbf{a}_T^H(\phi_u) \times e^{j2\pi f_{d,c,u} n T_{\text{sym}}}, \quad (1)$$

where $\alpha_{c,u}$, $\tau_{c,u}$, and $f_{d,c,u}$ denote the complex path gain, the propagation delay, and the Doppler shift of the u -th UE, respectively. f_{samp} is the system sampling rate, and $T_{\text{sym}} = (M + N_{\text{CP}})T_s$ represents the OFDM symbol duration with N_{CP} being the length of the cyclic prefix (CP). $\mathbf{a}_R(\theta_u)$ and $\mathbf{a}_T(\phi_u)$ are the receive and transmit array steering vectors, respectively, which are defined as $\mathbf{a}_R(\theta_u) = [1, e^{-j2\pi \frac{d}{\lambda_c} \sin \theta_u}, \dots, e^{-j2\pi \frac{d(N_{\text{BS}}-1)}{\lambda_c} \sin \theta_u}]^T$ and $\mathbf{a}_T(\phi_u) = [1, e^{-j2\pi \frac{d}{\lambda_c} \sin \phi_u}, \dots, e^{-j2\pi \frac{d(N_{\text{UE}}-1)}{\lambda_c} \sin \phi_u}]^T$, where d is the adjacent antenna spacing, λ_c is the wavelength of carrier, and θ_u and ϕ_u represent the angle of arrival (AoA) and the angle of departure (AoD) of the u -th UE, respectively.

For sensing, the frequency-domain matrix of sensing channel for the k -th point target at the m -th subcarrier and the n -th OFDM symbol can be modeled as

$$\mathbf{G}_k^m(n) = \alpha_{s,k} e^{-j2\pi\tau_{s,k} f_{\text{samp}} \frac{m}{M}} \mathbf{a}_R(\theta_k) \mathbf{a}_T^T(\phi_k) \times e^{j2\pi f_{d,s,k} n T_{\text{sym}}}, \quad (2)$$

where $\alpha_{s,k}$, $\tau_{s,k}$, and $f_{d,s,k}$ denote the complex reflection coefficient, the round-trip time delay, and the Doppler shift of the k -th target, respectively. f_{samp} is the system sampling rate, and $T_{\text{sym}} = (M + N_{\text{CP}})T_s$ represents the OFDM symbol duration. The receive and transmit array steering vectors of the BS, denoted by $\mathbf{a}_R(\theta_k)$ and $\mathbf{a}_T(\phi_k)$, are given by $\mathbf{a}_R(\theta_k) = [1, e^{-j2\pi \frac{d}{\lambda_c} \sin \theta_k}, \dots, e^{-j2\pi \frac{d(N_{\text{BS}}-1)}{\lambda_c} \sin \theta_k}]^T$ and $\mathbf{a}_T(\phi_k) = [1, e^{-j2\pi \frac{d}{\lambda_c} \sin \phi_k}, \dots, e^{-j2\pi \frac{d(N_{\text{BS}}-1)}{\lambda_c} \sin \phi_k}]^T$, respectively. It should be noted that for the k -th target, the azimuth AoD ϕ_k and AoA θ_k are identical, i.e., $\phi_k = \theta_k$, which is due to the fact that the BS transmit and receive arrays are co-located.

B. Signal Model

At the BS, the total received signal is the combination of the uplink communication signals from U active UEs and the downlink sensing echoes reflected by K targets. We define $\mathbf{s}_{c,u}^m(n) = \mathbf{P}_{\text{RF},u}^{\text{UE}} \mathbf{P}_{\text{BB},u}^m(n) \mathbf{x}_{c,u}^m(n)$ and $\mathbf{s}_s^m(n) = \mathbf{P}_{\text{RF}}^{\text{BS}} \mathbf{P}_{\text{BB}}^m(n) \mathbf{x}_s^m(n)$ as the precoded transmitted signals for the u -th UE and the ISAC BS, respectively, where $\mathbf{x}_{c,u}^m(n)$ and $\mathbf{x}_s^m(n)$ are the corresponding information symbols. By processing the aggregate signal through an equivalent receiver combiner $\mathbf{W}^m = \mathbf{W}_{\text{RF}}^{\text{BS}} \mathbf{W}_{\text{BB}}^m$, the total received signal at the m -th subcarrier and n -th OFDM symbol can be expressed as

$$\mathbf{y}^m(n) = \sum_{u=1}^U (\mathbf{W}^m)^H \mathbf{H}_u^m(n) \mathbf{s}_{c,u}^m(n) + \sum_{k=1}^K (\mathbf{W}^m)^H \mathbf{G}_k^m(n) \mathbf{s}_s^m(n) + \mathbf{z}^m(n), \quad (3)$$

where $\mathbf{z}^m(n) \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$ denotes the additive white Gaussian noise (AWGN).

III. UNIFIED TENSOR FORMULATION

The signal matrix $\mathbf{Y}^m = [\mathbf{y}^m(1), \dots, \mathbf{y}^m(N)] \in \mathbb{C}^{M_{\text{BS}} \times N}$ is constructed at the m -th subcarrier by stacking the received signals across N symbols. The received signal matrix $\mathbf{Y}^m$ can be reformulated as

$$\mathbf{Y}^m = \sum_{i=1}^L e^{-j2\pi\tau_i f_{\text{samp}} \frac{m}{M}} \tilde{\mathbf{a}}_R(\alpha_i, \theta_i) \tilde{\mathbf{a}}_T^T(\phi_i) \mathbf{\Gamma}(f_{d,i}) + \mathbf{Z}^m, \quad (4)$$

where $L = U + K$ denotes the total number of spatial paths, which comprises both communication UEs and radar targets, and τ_i is the time delay associated with the i -th path. The term $e^{-j2\pi\tau_i f_{\text{samp}} \frac{m}{M}}$ represents the phase shift induced by the signal delay at the m -th subcarrier. The equivalent received array steering vector is defined as $\tilde{\mathbf{a}}_R(\alpha_i, \theta_i) = \alpha_i \mathbf{W}^H \mathbf{a}_R(\theta_i) \in \mathbb{C}^{M_{\text{BS}} \times 1}$, and the equivalent transmitted vector is $\tilde{\mathbf{a}}_T(\phi_i) = \mathbf{S}_i^T \mathbf{a}_T(\phi_i) \in \mathbb{C}^{N \times 1}$, where $\mathbf{S}_i$ denotes the stacked transmitted symbol sequence. The diagonal Doppler shift matrix is given by $\mathbf{\Gamma}(f_{d,i}) = \text{diag}([e^{j2\pi f_{d,i} T_{\text{sym}}}, \dots, e^{j2\pi f_{d,i} N T_{\text{sym}}}]^T) \in \mathbb{C}^{N \times N}$, and $\mathbf{Z}^m \in \mathbb{C}^{M_{\text{BS}} \times N}$ represents the AWGN.

To effectively exploit the multi-dimensional structure of the ISAC signals, the received matrices are stacked across all M

subcarriers to construct a third-order observation tensor $\mathcal{Y} \in \mathbb{C}^{M_{BS} \times N \times M}$. Under the adopted pilot and combiner design, the received tensor admits the following CPD representation:

$$\mathcal{Y} = [\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}] + \mathcal{Z} = \sum_{i=1}^L \mathbf{a}_i^{(1)} \circ \mathbf{a}_i^{(2)} \circ \mathbf{a}_i^{(3)} + \mathcal{Z}, \quad (5)$$

where $\mathcal{Z}$ is the noise tensor. The factor matrix $\mathbf{A}^{(n)} = [\mathbf{a}_1^{(n)}, \dots, \mathbf{a}_L^{(n)}]$ for $n \in \{1, 2, 3\}$ gathers the latent components along the spatial, temporal, and frequency domains, the dimensions of which are $M_{BS} \times L$, $N \times L$, and $M \times L$, respectively. Specifically, their column vectors are defined as $\mathbf{a}_i^{(1)} = \tilde{\mathbf{a}}_R(\alpha_i, \theta_i)$ indicating the spatial reception profile, $\mathbf{a}_i^{(2)} = \Gamma(f_{d,i})\tilde{\mathbf{a}}_T(\phi_i)$ accounting for both the Doppler shift and the transmit spatial steering, and $\mathbf{a}_i^{(3)} = [e^{-j2\pi\tau_i f_{samp} \frac{M}{N}}, \dots, e^{-j2\pi\tau_i f_{samp} \frac{M}{N}}]^T$ denoting the delay-dependent phase progression across subcarriers.

The exact CP-rank of the noiseless observation tensor corresponds directly to the total number of spatial paths, i.e., $L = U + K$. To ensure the identifiability of these L paths, traditional tensor decomposition algorithms require the rank L to be known *a priori* and Kruskal's uniqueness condition is required. However, the exact number of UEs and targets may dynamically fluctuate and remain unknown to the BS receiver in practice. To solve this problem, a VB-CPD factorization method is proposed, which incorporates a sparsity-inducing prior over the latent multi-linear factors, which enables appropriate estimation of tensor rank L , as well as the channel and target parameters.

IV. PROPOSED HIERARCHICAL VARIATIONAL BAYESIAN FRAMEWORK

A. Hierarchical Bayesian Model

To guarantee efficient parameter estimation with accurate rank inference even in noisy environment, the CPD model is transformed into a Bayesian probabilistic framework. Given that the observed tensor $\mathcal{Y} = \mathcal{X} + \mathcal{Z}$ is corrupted by independent and identically distributed (i.i.d.) zero-mean complex Gaussian noise $\mathcal{Z}$, the likelihood function is formulated as

$$p(\mathcal{Y} | \{\mathbf{A}^{(n)}\}_{n=1}^3, \beta) = \mathcal{CN}\left(\mathcal{Y} \middle| \sum_{i=1}^L \mathbf{a}_i^{(1)} \circ \mathbf{a}_i^{(2)} \circ \mathbf{a}_i^{(3)}, \beta^{-1} \mathbf{I}\right), \quad (6)$$

where $\mathbf{a}_i^{(n)}$ represents the i -th column vector of the factor matrix $\mathbf{A}^{(n)}$, and β denotes the noise precision.

Different from the deterministic methods relying on pre-defined ranks, a sparsity-inducing hierarchical prior over the factor matrices is defined, alongside Gamma hyperpriors for the precision parameters, given by

$$\begin{cases} p(\mathbf{A}^{(n)} | \boldsymbol{\omega}) = \prod_{j=1}^{I_n} \mathcal{CN}(\mathbf{a}_j^{(n)} | \mathbf{0}, \boldsymbol{\Omega}^{-1}), & \forall n \in \{1, 2, 3\}, \\ p(\boldsymbol{\omega}) = \prod_{i=1}^L \text{Ga}(\omega_i | c_0, d_0), \\ p(\beta) = \text{Ga}(\beta | \epsilon_0, \delta_0), \end{cases} \quad (7)$$

where I_n is the dimension of the n -th mode, $\mathbf{a}_j^{(n)}$ is the j -th row of $\mathbf{A}^{(n)}$, and $\boldsymbol{\Omega} = \text{diag}(\boldsymbol{\omega})$ is the precision matrix shared across all modes. The hyperparameter vector $\boldsymbol{\omega} = [\omega_1, \dots, \omega_L]^T$ controls the column-wise sparsity, while c_0, d_0, ϵ_0 , and δ_0 are set as small non-informative parameters.

Define the comprehensive parameter set as $\Psi = \{\{\mathbf{A}^{(n)}\}_{n=1}^3, \boldsymbol{\omega}, \beta\}$, the joint distribution of the observed tensor $\mathcal{Y}$ and the latent parameters Ψ is formulated as

$$p(\mathcal{Y}, \Psi) = p(\mathcal{Y} | \{\mathbf{A}^{(n)}\}_{n=1}^3, \beta) \left(\prod_{n=1}^3 p(\mathbf{A}^{(n)} | \boldsymbol{\omega}) \right) p(\boldsymbol{\omega}) p(\beta). \quad (8)$$

This generative model effectively balances data-driven multi-linear fidelity with structural sparsity constraints.

B. Variational Bayesian Inference

Since the exact posterior inference involves intractable calculation of marginal likelihood, the Variational Bayesian (VB) framework is considered. The objective is to find a deterministic approximation $q(\Psi)$ by minimizing the Kullback-Leibler (KL) divergence, which is mathematically equivalent to maximizing the Evidence Lower Bound (ELBO). With the mean-field approximation, the variational distribution factorizes over the distinct parameter groups, expressed as

$$q(\Psi) = q_\beta(\beta) q_\omega(\boldsymbol{\omega}) \prod_{n=1}^3 q_n(\mathbf{A}^{(n)}). \quad (9)$$

The optimal distribution for each parameter subset $q_j(\Psi_j)$ can be iteratively updated by taking the expectation of the log-joint distribution with respect to all other variables, expressed as

$$\ln q_j(\Psi_j) = \mathbb{E}_{q(\Psi \setminus \Psi_j)} [\ln p(\mathcal{Y}, \Psi)] + \text{const}. \quad (10)$$

Given the conjugate priors, the optimal functional forms for all variational factors can be derived analytically. This coordinate ascent process ensures monotonic convergence of the ELBO, enabling robust parameter tracking without requiring prior knowledge of the tensor rank.

C. Variational Posterior Updates

With the mean-field approximation, the optimal variational posterior $q_n(\mathbf{A}^{(n)})$ follows a factorized Matrix Complex Gaussian distribution, where each row $\mathbf{a}_{j,\cdot}^{(n)} \sim \mathcal{CN}(\mathbb{E}[\mathbf{a}_{j,\cdot}^{(n)}], \boldsymbol{\Sigma}^{(n)})$. The posterior covariance $\boldsymbol{\Sigma}^{(n)} \in \mathbb{C}^{L_{max} \times L_{max}}$ and expectation $\mathbb{E}[\mathbf{A}^{(n)}] \in \mathbb{C}^{I_n \times L_{max}}$ are analytically updated as

$$\begin{aligned} \boldsymbol{\Sigma}^{(n)} &= (\mathbb{E}[\beta] \underset{m \neq n}{\otimes} \mathbb{E} \left[\left((\mathbf{A}^{(m)})^H \mathbf{A}^{(m)} \right)^* \right] \\ &\quad + \text{diag}(\mathbb{E}[\boldsymbol{\omega}]))^{-1}, \end{aligned} \quad (11)$$

$$\mathbb{E}[\mathbf{A}^{(n)}] = \mathbb{E}[\beta] \mathbf{Y}_{(n)} \mathbb{E}[\mathbf{A}_{\setminus n}^*] \boldsymbol{\Sigma}^{(n)}. \quad (12)$$

where L_{max} is the initialized rank upper bound.

The variational posteriors $q(\omega_i)$ and $q(\beta)$ follow Gamma distributions, with their expectations updated as $\mathbb{E}[\omega_i] = \tilde{c}_i / \tilde{d}_i$ and $\mathbb{E}[\beta] = \tilde{\epsilon} / \tilde{\delta}$, respectively. Specifically, the parameters are computed by $\tilde{c}_i = c_0 + \sum_{n=1}^3 I_n$, $\tilde{d}_i = d_0 +$

$$\sum_{n=1}^3 \mathbb{E} \left[\|\mathbf{a}_{:,i}^{(n)}\|_2^2 \right], \tilde{\epsilon} = \epsilon_0 + \prod_{n=1}^3 I_n, \text{ and } \tilde{\delta} = \delta_0 + \mathbb{E} \left[\|\mathcal{Y} - [\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}]\|_F^2 \right].$$

The latent variables $\{\mathbf{A}^{(n)}\}_{n=1}^3$, $\boldsymbol{\omega}$, and β are iteratively updated until the ELBO converges. Upon convergence, redundant components characterized by $\mathbb{E}[\omega_i] = \infty$ are discarded, thereby automatically determining the effective tensor rank $\hat{L}$. Consequently, the factor matrices are directly extracted from their remaining active columns, expressed as $\hat{\mathbf{A}}^{(n)} = \mathbb{E}[\mathbf{A}^{(n)}]_{:,1:\hat{L}}$ for all $n \in \{1, 2, 3\}$, which can be used to extract the subsequent physical parameter.

V. PHYSICAL PARAMETER EXTRACTION VIA ALTERNATING DECOUPLING

Given the estimated factor matrices $\hat{\mathbf{A}}^{(1)} \in \mathbb{C}^{M_{BS} \times \hat{L}}$, $\hat{\mathbf{A}}^{(2)} \in \mathbb{C}^{N \times \hat{L}}$, and $\hat{\mathbf{A}}^{(3)} \in \mathbb{C}^{M \times \hat{L}}$ from the Bayesian inference stage, the physical parameters $\{\theta_i, \tau_i, \phi_i, f_{d,i}\}$ for each active path $i = 1, \dots, \hat{L}$ can be extracted. Let $\hat{\mathbf{a}}_i^{(1)}$ and $\hat{\mathbf{a}}_i^{(3)}$ denote the i -th column of $\hat{\mathbf{A}}^{(1)}$ and $\hat{\mathbf{A}}^{(3)}$, respectively. The AoA θ_i and propagation delay τ_i are independently estimated via 1D correlation-based peak searches, expressed as

$$\begin{cases} \hat{\theta}_i = \arg \max_{\theta} \frac{\left| (\hat{\mathbf{a}}_i^{(1)})^H \mathbf{W}^H \mathbf{a}_R(\theta) \right|^2}{\|\hat{\mathbf{a}}_i^{(1)}\|_2^2 \|\mathbf{W}^H \mathbf{a}_R(\theta)\|_2^2}, \\ \hat{\tau}_i = \arg \max_{\tau} \frac{\left| (\hat{\mathbf{a}}_i^{(3)})^H \mathbf{c}(\tau) \right|^2}{\|\hat{\mathbf{a}}_i^{(3)}\|_2^2 \|\mathbf{c}(\tau)\|_2^2}. \end{cases} \quad (13)$$

Since the AoD ϕ_i and Doppler shift $f_{d,i}$ are coupled with the transmitted signal sequences in $\hat{\mathbf{a}}_i^{(2)}$, the extraction process of ϕ_i and $f_{d,i}$ can be solved by a non-linear least-squares (LS) optimization, given by

$$\min_{\delta_i, f_{d,i}, \mathbf{S}_i, \phi_i} \left\| \hat{\mathbf{a}}_i^{(2)} - \delta_i \mathbf{\Gamma}(f_{d,i}) \mathbf{S}_i^T \mathbf{a}_T(\phi_i) \right\|_2^2. \quad (14)$$

For notational simplicity, $\mathbf{a}_T(\phi_i)$ and $\mathbf{S}_i$ are treated as path-dependent quantities in the APD procedure, denoting the UE transmit steering vector and pilot/precoding sequence for communication components and the BS transmit steering vector and sensing waveform for sensing components. The component type is identified from the known pilot/waveform dictionaries, and $\phi_i = \theta_i$ is imposed for sensing paths. To avoid local optimum, the initial $\hat{\mathbf{S}}_i^{(0)}$ and AoD $\hat{\phi}_i^{(0)}$ are obtained via a 2D joint search with $\hat{f}_{d,i}^{(0)} = 0$, given by

$$\{\hat{\mathbf{S}}_i^{(0)}, \hat{\phi}_i^{(0)}\} = \arg \max_{\mathbf{S}_i \in \mathcal{S}, \phi} \frac{\left| \mathbf{a}_T^H(\phi) \mathbf{S}_i^* \hat{\mathbf{a}}_i^{(2)} \right|^2}{\|\mathbf{S}_i^T \mathbf{a}_T(\phi)\|_2^2}. \quad (15)$$

In the j -th iteration, by fixing other variables, the parameters are sequentially updated to decrease the LS cost function. More specifically, the scaling factor $\hat{\delta}_i^{(j)}$ is updated via its closed-form solution, given by

$$\hat{\delta}_i^{(j)} = \frac{\mathbf{a}_T^H(\hat{\phi}_i^{(j-1)}) (\hat{\mathbf{S}}_i^{(j-1)})^* \mathbf{\Gamma}^{-1}(\hat{f}_{d,i}^{(j-1)}) \hat{\mathbf{a}}_i^{(2)}}{\|(\hat{\mathbf{S}}_i^{(j-1)})^T \mathbf{a}_T(\hat{\phi}_i^{(j-1)})\|_2^2}. \quad (16)$$

With fixed $\hat{\delta}_i^{(j)}$, the Doppler shift is decoupled and estimated via a 1D spectral peak search, expressed as

$$\hat{f}_{d,i}^{(j)} = \arg \max_{f_d} \sum_{n=1}^N \text{Re} \left\{ e^{j2\pi f_d n T_{sym}} \hat{\delta}_i^{(j)} [\hat{\mathbf{a}}_i^{(2)}]_n^* \times \left(\hat{\mathbf{s}}_i^{(j-1)}(n) \right)^T \mathbf{a}_T(\hat{\phi}_i^{(j-1)}) \right\}, \quad (17)$$

where $[\hat{\mathbf{a}}_i^{(2)}]_n$ denotes the n -th element of $\hat{\mathbf{a}}_i^{(2)}$. Finally, the sequence and AoD are updated by maximizing their respective spatial-signal correlations, given by

$$\begin{cases} \hat{\mathbf{S}}_i^{(j)} = \arg \max_{\mathbf{S}_i \in \mathcal{S}} \frac{\left| \mathbf{a}_T^H(\hat{\phi}_i^{(j-1)}) \mathbf{S}_i^* \mathbf{\Gamma}^{-1}(\hat{f}_{d,i}^{(j)}) \hat{\mathbf{a}}_i^{(2)} \right|^2}{\|\mathbf{S}_i^T \mathbf{a}_T(\hat{\phi}_i^{(j-1)})\|_2^2}, \\ \hat{\phi}_i^{(j)} = \arg \max_{\phi} \frac{\left| \mathbf{a}_T^H(\phi) (\hat{\mathbf{S}}_i^{(j)})^* \mathbf{\Gamma}^{-1}(\hat{f}_{d,i}^{(j)}) \hat{\mathbf{a}}_i^{(2)} \right|^2}{\|(\hat{\mathbf{S}}_i^{(j)})^T \mathbf{a}_T(\phi)\|_2^2}. \end{cases} \quad (18)$$

VI. SIMULATION RESULTS

In this section, simulation results are presented to verify the effectiveness of the proposed VB-CPD algorithm. We assume the carrier frequency is 60GHz, the BS is equipped with a ULA of $N_{BS} = 64$ antennas and $M_{BS} = 40$ RF chains. There are $U = 4$ UEs and $K = 4$ targets, yielding a total of $L = 8$ spatial paths to be resolved. Each UE is equipped with $N_{UE} = 32$ antennas and a single RF chain. For OFDM, the number of subcarriers is set to 256, among which $M = 60$ subcarriers are allocated for pilot transmissions, and the sampling rate of 200MHz. A temporal observation window of $N = 80$ OFDM symbols is accumulated to construct the observation tensor. To quantitatively evaluate the estimation accuracy, the Normalized Mean Square Error (NMSE) is adopted as the performance metric, given by

$$\text{NMSE} = \frac{1}{N_{mc}} \sum_{n=1}^{N_{mc}} \frac{\|\hat{\Theta}^{(n)} - \Theta\|^2}{\|\Theta\|^2}, \quad (19)$$

where N_{mc} denotes the total number of independent Monte Carlo trials, Θ represents the ground-truth parameter considering the aggregated channel matrix, spatial angles, and time delays, $\hat{\Theta}^{(n)}$ is the corresponding estimated value.

Fig. 1 presents the rank estimation performance, where the true tensor rank is fixed with $L = 8$. It can be observed that, the estimation error rate decreases significantly with the increase of signal-to-noise ratio (SNR), which is only 5% when SNR is 20 dB. This indicates that the proposed VB-CPD algorithm achieves a highly reliable rank estimation success rate in the high signal-to-noise ratio regime, thus successfully bypassing the severe rank-mismatch degradation inherently faced by traditional fixed-rank methods.

Fig. 2 evaluates the overall channel matrix estimation NMSE. The proposed VB-CPD consistently outperforms the baseline methods. It can be observed that, the conventional ALS algorithm exhibits the worst performance due to the rank mismatch. The Vandermonde method and VB-CPD achieve a comparable NMSE performance when 0 dB, and the proposed

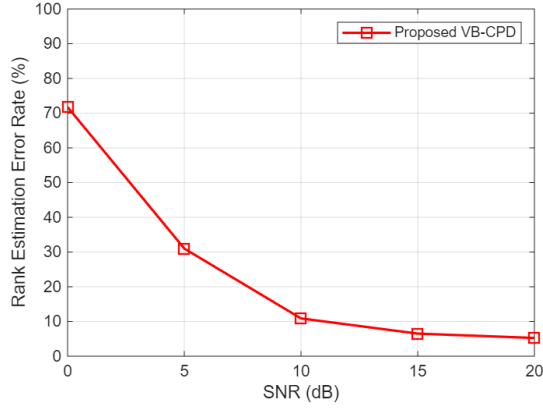


Fig. 1. Rank estimation performance.

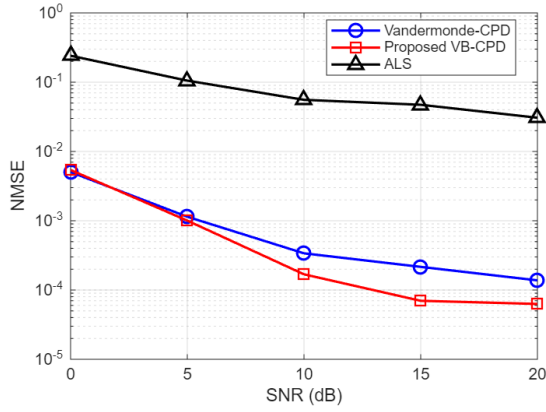


Fig. 2. Communication channel estimation performance.

VB-CPD steadily converges to a significantly lower error floor when SNR is high, which is better than Vandermonde method, thus demonstrating the outstanding performance of the proposed method.

Fig. 3 illustrates the sensing parameter estimation performance. As for the time delay, the estimation error of the proposed VB-CPD method remarkably stabilizes near 2×10^{-8} , which is substantially lower than that produced by the Vandermonde-CPD method. In addition, the proposed algorithm demonstrates outstanding resolution in terms of spatial domain. For example, the AoA estimation NMSE achieved by VB-CPD is 1.5×10^{-6} at 0 dB, which outperforms the Vandermonde-CPD and ALS algorithms. Furthermore, the AoD estimation NMSE for the VB-CPD decreases sharply from 10^{-3} to 5×10^{-7} , whereas the other baseline schemes suffer from much higher error floors. Additionally, the Doppler shift estimation performance of the proposed method also outperforms other schemes. These above results comprehensively verify the superiority of the proposed scheme.

VII. CONCLUSION

In this paper, a robust parameter estimation framework was proposed for mmWave ISAC system. By establishing

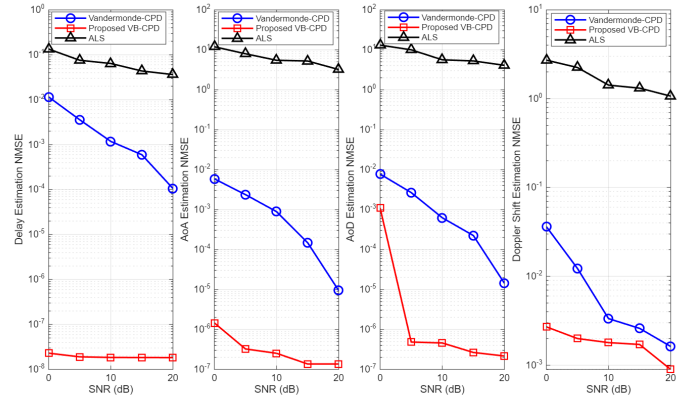


Fig. 3. Sensing parameters estimation performance.

a unified multi-dimensional tensor model, we developed a VB-CPD algorithm that automatically determines the tensor rank. Furthermore, an APD algorithm was introduced to sequentially resolve the highly non-linear coupling among spatial angles, time delays, and Doppler shifts. Simulation results demonstrated that the proposed framework substantially outperforms conventional tensor decomposition baselines in both rank identification and parameter extraction.

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